



2 Mathematics

Supplement Package

Teacher Resource

Binder

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Introduction

The *Georgia Math 2 Supplement Package* provides a complete set of resources to complement any Math 2 instructional program. Each activity and tool addresses Georgia's Math 2 Framework and supports achievement of Georgia's Performance Standards.

The *Georgia Math 2 Supplement Package* includes components that encourage best practice instruction. The package provides:

- Graphic Organizers to support a range of learning styles and to foster higher-level thinking
- Pre- and Post-Assessments to inform instruction and gauge achievement
- Warm-Up Activities to engage students and activate prior knowledge with Debrief Notes to make connections to the lesson of the day
- Station Activities to allow students to explore and apply the mathematics they are learning through hands-on, small-group problem solving

UNIT • QUADRATICS AND COMPLEX NUMBERS**Lesson 1: Analyzing Quadratics in Vertex Form $f(x) = (x - h)^2$** **Pre-Assessment**

Circle the best answer.

1. The solutions of the equation $0 = (x + 2)(x - 3)$ are:

a. $x = 2, -3$

c. $x = -2, -3$

b. $x = 2, 3$

d. $x = -2, 3$

2. Expand the following expression:

$$(x + 3)^2 =$$

a. $x^2 + 9$

c. $x^2 + 3x + 9$

b. $x^2 + 6$

d. $x^2 + 6x + 9$

3. Simplify the following expression:

$$\frac{1}{2} + \frac{2}{3} + \frac{3}{4}$$

a. $\frac{6}{9}$

c. $\frac{23}{12}$

b. $\frac{2}{3}$

d. $\frac{12}{23}$

4. Find the solution set of $4x - 3 > 5$.

a. $x < 4$

c. $x < 8$

b. $x > 2$

d. $x > 8$

5. Perform the following operation:

$$(2\sqrt{2} + 3) + (-2 + \sqrt{8})$$

a. $4\sqrt{2} + 1$

c. $4\sqrt{2} + 5$

b. $\sqrt{8} + \sqrt{2} + 1$

d. $2\sqrt{2} + 3$

UNIT • QUADRATICS AND COMPLEX NUMBERS**Lesson 1: Analyzing Quadratics in Vertex Form $f(x) = (x - h)^2$** **Assessment****Post-Assessment**

Circle the best answer.

- Given the graph of $y = x^2$, to get the graph of $y = (x - 3)^2$, translate the graph of $y = x^2$:
 - 3 units to the left
 - 3 units to the right
 - 3 units up
 - 3 units down
- Given the graph of $y = x^2$, to get the graph of $y = (x + 2)^2$, translate the graph of $y = x^2$:
 - 2 units to the left
 - 2 units to the right
 - 2 units up
 - 2 units down
- Given the graph of $y = (x + 1)^2$, to get the graph of $y = (x + 5)^2$, translate the graph of $y = (x + 1)^2$:
 - 4 units to the left
 - 4 units to the right
 - 5 units to the left
 - 5 units to the right
- The graph of the function $f(x)$ is moved 4 units to the left. What is the equation of the new function?
 - $y = f(x - 4)$
 - $y = f(x + 4)$
 - $y = f(x) + 4$
 - $y = f(x) - 4$
- The graph of the function $g(x)$ is moved 3 units to the right and 1 unit down. What is the equation of the new function?
 - $y = g(x - 3) + 1$
 - $y = g(x) - 3 + 1$
 - $y = g(x + 3) - 1$
 - $y = g(x - 3) - 1$

continued

UNIT • QUADRATICS AND COMPLEX NUMBERS**Lesson 1: Analyzing Quadratics in Vertex Form $f(x) = (x - h)^2$** **Assessment**

6. What are the solutions to the equation $x^2 - 3x - 40 = 0$?
- a. $x = -8$ only
b. $x = 5$ only
c. $x = -8$ and 5
d. $x = -5$ and 8
7. What are the solutions to the equation $x^2 - 2x - 120 = -57$?
- a. $x = -10$ and 12
b. $x = -12$ and 10
c. $x = -7$ and 9
d. $x = -7$ and -9
8. What are the solutions to the equation $(x + 3)^2 - 4 = 21$?
- a. $x = 22$ only
b. $x = -22$ only
c. $x = -8$ and -5
d. $x = -8$ and 2
9. What are the solutions to the equation $(2x - 4)^2 + 20 = 84$?
- a. $x = -2$ only
b. $x = 6$ only
c. $x = -2$ and 6
d. $x = 2$ and 6
10. What are the solutions to the equation $(-x + 2)^2 - 21 = 60$?
- a. $x = -7$ and 11
b. $x = -11$ and 7
c. $x = -11$ and -7
d. $x = 7$ and 11
11. Describe the transformation of the function $y = -(x + 4)^2 + 2$ compared to the parent function of $y = x^2$.

Pre-Assessment

Circle the best answer.

- The y -intercept of the equation $y = (x + 5)^2 - 9$ is:
 - 4
 - 2
 - 2
 - 16
- Simplify $(2x - 3)(x + 4)$.
 - $2x^2 + 5x - 12$
 - $2x^2 - 12$
 - $8x - 3x$
 - $3x + 1$
- Solve for x : $3x^2 + 9x = 0$
 - $x = -3$
 - $x = 0$
 - $x = 0, -3$
 - $x = 0, 3$
- A horizontal line and a parabola will
 - intersect at most three times.
 - intersect at most two times.
 - intersect at most once.
 - never intersect.
- The x -intercept of the equation $y = x^2 + 4x + 3$ is:
 - $x = -3$
 - $x = -3, -1$
 - $x = -3, 1$
 - $x = 3, 1$

Post-Assessment

Circle the best answer.

1. Solve for x : $x^2 + 8x + 15 = 0$

a. $x = -5$

c. $x = -3, -5$

b. $x = 3$

d. $x = 3, 5$

2. Solve for x : $x^2 + 3x - 18 = 0$

a. $x = -6, -3$

c. $x = -3, 6$

b. $x = -6, 3$

d. $x = 3, 6$

3. Solve for x : $x^2 + 40x = 0$

a. $x = -40$

c. $x = 40$

b. $x = 0$

d. $x = -40, 0$

4. Solve for x : $(x + 2)(x - 4)(x + 5) = 0$

a. $x = 2, -4, 5$

c. $x = -2, -4, -5$

b. $x = -2, 4, -5$

d. $x = 2, 4, 5$

5. Solve for x : $x(x + 6)(x - 3)(x + 9) = 0$

a. $x = -9, -6, 0, 3$

c. $x = 0, 3, 9$

b. $x = -3, 0, 3, 9$

d. $x = -9, -3, 0$

6. What is the next step you must take in order to solve $x^2 - 3x = 4$ for x ?

a. Set x and $x - 3$ equal to 4.

b. Take the square root of both sides.

c. Subtract 4 from both sides.

d. Divide both sides by x .

continued

UNIT • QUADRATICS AND COMPLEX NUMBERS**Lesson 2: Finding the y -intercept in Standard Form and Analyzing Graphs****Assessment**

7. A quadratic function has how many x -intercepts?
- a. none
 - b. always one
 - c. always two
 - d. at most two
8. A quadratic function has how many y -intercepts?
- a. none
 - b. always one
 - c. always two
 - d. at most two
9. To solve $(x + 3)(x - 2) = 0$, what property do you use?
- a. Zero-Product Property
 - b. Equality Property of Addition
 - c. Equality Property of Multiplication
 - d. Distributive Property
10. The graphs of two different quadratic functions will intersect
- a. on the x -axis.
 - b. on the y -axis.
 - c. in at most two places.
 - d. in at most three places.
11. How would you find the intersection of the graphs of two equations?

Pre-Assessment

Circle the best answer.

1. $(x + 3y)(2x - 4) =$

a. $2x^2 + 3y - 4$

c. $2x^2 + 12xy - 6y - 4$

b. $2x^2 - 12y^2 + 6xy$

d. $2x^2 + 6xy - 4x - 12y$

2. Solve for x : $x^2 - 3x - 28 = 0$

a. $x = -7, -4$

c. $x = -4, 7$

b. $x = -7, 4$

d. $x = 4, 7$

3. To move the graph of the function $f(x)$ up 5 units, graph:

a. $f(x) - 5$

c. $f(x + 5)$

b. $f(x) + 5$

d. $f(x - 5)$

4. What product is represented by the following diagram?

$3x^2$	$5x$
$6x$	10

a. $(3x + 5)(x + 2)$

c. $(3)(x^2 + 2x)$

b. $(6x)(10 + 5x)$

d. $(3x + 2)^2$

5. The x -intercept of an equation is when:

a. $x = 0$

c. $y = x$

b. $y = 0$

d. $y = 1$

Post-Assessment

Circle the best answer.

- The factors of $2x^2 - 5x - 12$ are:
 - $(2x + 3)(x + 4)$
 - $(2x + 3)(x - 4)$
 - $(2x - 3)(x + 4)$
 - $(2x - 3)(x - 4)$
- The factors of $3x^2 + 22x + 35$ are:
 - $(3x + 7)(x + 5)$
 - $(3x + 7)(x - 5)$
 - $(3x - 7)(x + 5)$
 - $(3x - 7)(x - 5)$
- The factors of $2x^2 - 24x + 54$ are:
 - $2(x + 9)(x + 3)$
 - $2(x + 9)(x - 3)$
 - $2(x - 9)(x + 3)$
 - $2(x - 9)(x - 3)$
- The factors of $-9x^2 + 18x + 16$ are:
 - $(3x + 2)(3x + 8)$
 - $(3x + 2)(3x - 8)$
 - $(-3x + 2)(3x - 8)$
 - $(3x + 2)(-3x + 8)$
- The factors of $150x^2 + 140x - 80$ are:
 - $10(5x - 2)(3x + 4)$
 - $10(5x + 2)(3x - 4)$
 - $(50x - 20)(3x + 4)$
 - $(50x + 20)(3x - 4)$
- Solve for x : $2x^2 - 4x - 6 = 0$
 - $x = -1$
 - $x = -3$
 - $x = -3, -1$
 - $x = -1, 3$

continued

NAME: _____

UNIT • QUADRATICS AND COMPLEX NUMBERS

Lesson 3: Factoring $ax^2 + bx + c = 0$



Assessment

7. Solve for x : $3x^2 - 3x - 60 = 0$

a. $x = -5$

c. $x = -4, 5$

b. $x = -4$

d. $x = -5, -4$

8. Solve for x : $2x^2 - 11x + 14 = 0$

a. $x = -4, -7$

c. $x = 2, \frac{7}{2}$

b. $x = -2, -\frac{7}{2}$

d. $x = 4, 7$

9. Solve for x : $6x^2 + 36x = -30$

a. $x = 1, 5$

c. $x = 6$

b. $x = -1, -5$

d. $x = -6$

10. Solve for x : $6x(x + 1) = 5(x + 3)$

a. $x = \frac{5}{3}, \frac{3}{2}$

c. $x = -\frac{5}{3}, -\frac{3}{2}$

b. $x = -\frac{5}{3}, \frac{3}{2}$

d. $x = \frac{5}{3}, -\frac{3}{2}$

11. What are the steps to solve quadratic equations?

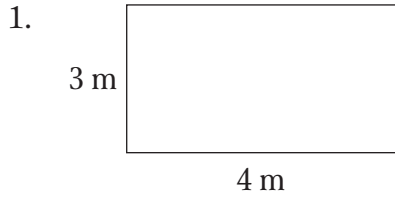
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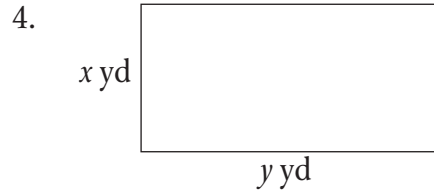
UNIT • QUADRATICS AND COMPLEX NUMBERS

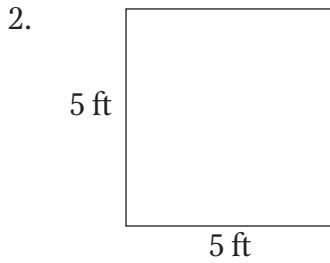
Lesson 1: Analyzing Quadratics in Vertex Form $f(x) = (x - h)^2$

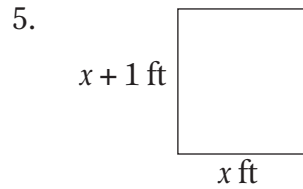
Warm-Up

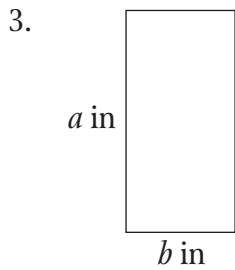
Quadratic functions are used to model areas of figures. Remember that if l is the length of a rectangle and w is its width, then the area of the rectangle is given by $A = lw$. Find the area of each of the following rectangles.

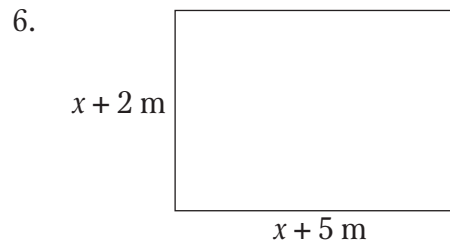












Warm-Up Debrief

Explain to students that problems 1 and 2 correspond to specific rectangles, but problems 3 through 6 correspond to many different rectangles. Ask students about what kinds of rectangles that the rectangle in problem 5 represents. Students might respond with “a rectangle of 4 feet and 5 feet” or some other specific example. Demonstrate that this rectangle represents any rectangle in which one side is one more foot than the other side.

Students prefer specific examples over the general equations for problems 3 through 6. On the board or overhead, draw some specific examples of the rectangle in problem 5. Start with any sides that the students suggest, or with 1 foot and 2 feet. Then increase both sides by 1 foot for 3 iterations (2 feet and 3 feet; 3 feet and 4 feet; 4 feet and 5 feet). Show that the equation for the area of the general rectangle in problem 5 matches the area of each of these specific examples.

NAME: _____

UNIT • QUADRATICS AND COMPLEX NUMBERS

Lesson 2: Finding the y -intercept in Standard Form and Analyzing Graphs



Warm-Up

Solve each equation for when $y = 0$, and then for when $x = 0$.

1. $y = 2x + 6$

2. $y = x^2$

3. $y = \frac{1}{3}x + 5$

4. $y = 5x$

5. $y = x^2 + 2x + 1$

6. $y = (3x + 5)(x - 2)$

7. $y = x(x - 1)(x - 3)$

8. $y = \frac{x}{x + 2}$

9. $y = \frac{x - 3}{x - 1}$

10. $y = x^2 + 7x + 12$

11. What do the points you found in problems 1 through 10 represent? Explain.

Warm-Up Debrief

Explain to students that when $x = 0$, the corresponding points on the graph of the equation lie on the y -axis. When $y = 0$, the corresponding points on the graph of the equation lie on the x -axis. These points are known as the y - and x -intercept, respectively. These points have important implications in applications.

UNIT • QUADRATICS AND COMPLEX NUMBERS**Lesson 3: Factoring $ax^2 + bx + c = 0$** **Warm-Up**

Writing the factors of a number is an important step in simplifying and adding fractions and factoring polynomials. Remember the factors of a number are all the numbers that divide it evenly. This includes 1 and the number itself. Note that all factors of a number are between 1 and the number itself. Below is an example of how to find all the factors of a number quickly.

Example

Write the positive factors of 36.

1. To write the factors of 36, start by asking “1 times what is 36?”

$$36 \cdot 1 = 36$$

2. Now ask, “2 times what is 36?”

$$18 \cdot 2 = 36$$

3. Move on to the next number and repeat the pattern.

$$12 \cdot 3 = 36$$

$$9 \cdot 4 = 36$$

4. Since 5 does not divide 36, move on to the next number.

$$6 \cdot 6 = 36$$

It is not necessary to continue since the next factor is 9, which is already shown above.

The factors of 36 are 1, 2, 3, 4, 6, 9, 12, 18, and 36.

Find all the positive factors of each number that follows.

1. 8

2. 27

3. 20

continued

NAME: _____

UNIT • QUADRATICS AND COMPLEX NUMBERS



Lesson 3: Factoring $ax^2 + bx + c = 0$

4. 30

5. 56

6. 24

7. 100

8. 50

9. 81

10. 144

11. Explain how you know when you have found all the possible positive factors of a number.

Warm-Up Debrief

Explain to students that to solve quadratic equations of the form $ax^2 + bx + c = 0$, first factor the left side of the equation. Each of those factors are then set equal to zero and solved for x . To do this for more complicated quadratic equations, students need to be able to write all the factors of a number.

Station Activities Guide

Introduction

Most units include a collection of station-based activities to provide students with opportunities to practice and apply the mathematical skills and concepts they are learning. You may use these activities in addition to the instructional lessons, or, especially if the pre-test or other formative assessment suggests it, instead of direct instruction in areas where students have the basic concepts but need practice. The debriefing discussions after each set of activities provide an important opportunity to help students reflect on their experiences and synthesize their thinking. It also provides an additional opportunity for ongoing, informal assessment to guide instructional planning.

Implementation Guide

The following guidelines will help you prepare for and use the activity sets in this section.

Setting Up the Stations

Each activity set consists of four stations. Set up each station at a desk, or at several desks pushed together, with enough chairs for a small group of students. Place a card with the number of the station on the desk. Each station should also contain the materials specified in the teacher's notes, and a stack of Student Activity Sheets (one copy per student). Place the required materials (as listed) at each station.

When a group of students arrives at a station, each student should take one of the activity sheets to record the group's work. Although students should work together to develop one set of answers for the entire group, each student should record the answers on his or her own activity sheet. This helps keep students engaged in the activity and gives each student a record of the activity for future reference.

Forming Groups of Students

All activity sets consist of four stations. You might divide the class into four groups by having students count off from 1 to 4. If you have a large class and want to have students working in small groups, you might set up two identical sets of stations, labeled A and B. In this way, the class can be divided into eight groups, with each group of students rotating through the "A" stations or "B" stations.

Assigning Roles to Students

Students often work most productively in groups when each student has an assigned role. You may want to assign roles to students when they are assigned to groups and change the roles occasionally. Some possible roles are as follows:

- Reader—reads the steps of the activity aloud
- Facilitator—makes sure that each student in the group has a chance to speak and pose questions; also makes sure that each student agrees on each answer before it is written down
- Materials Manager—handles the materials at the station and makes sure the materials are put back in place at the end of the activity
- Timekeeper—tracks the group’s progress to ensure that the activity is completed in the allotted time
- Spokesperson—speaks for the group during the debriefing session after the activities

Timing the Activities

The activities in this section are designed to take approximately 10 minutes per station. Therefore, you might plan on having groups change stations every 10 minutes, with a two-minute interval for moving from one station to the next. It is helpful to give students a “5-minute warning” before it is time to change stations.

Since each activity set consists of four stations, the above timeframe means that it will take about 50 minutes for groups to work through all stations.

Guidelines for Students

Before starting the first activity set, you may want to review the following “ground rules” with students. You might also post the rules in the classroom.

- All students in a group should agree on each answer before it is written down. If there is a disagreement within the group, discuss it with one another.
- You can ask your teacher a question only if everyone in the group has the same question.
- If you finish early, work together to write problems of your own that are similar to the ones on the Student Activity Sheet.
- Leave the station exactly as you found it. All materials should be in the same place and in the same condition as when you arrived.

Debriefing the Activities

After each group has rotated through every station, bring students together for a brief class discussion. At this time you might have the groups' spokespersons pose any questions they had about the activities. Before responding, ask if students in other groups encountered the same difficulty or if they have a response to the question. The class discussion is also a good time to reinforce the essential ideas of the activities. The questions that are provided in the teacher's notes for each activity set can serve as a guide to initiating this type of discussion.

You may want to collect the Student Activity Sheets before beginning the class discussion. However, it can be beneficial to collect the sheets afterward so that students can refer to them during the discussion. This also gives students a chance to revisit and refine their work based on the debriefing session. If you run out of time to hold class discussions, you might want to have students journal about their experiences and follow up with a class discussion the next day.

Instruction

Goal: To provide opportunities for students to analyze the relationship between the equation of a parabola and its graph

Georgia Performance Standard

MM2A3: Students will analyze quadratic functions in the forms $f(x) = ax^2 + bx + c$ and $f(x) = a(x - h)^2 + k$.

- a. Convert between standard and vertex form.
- b. Graph quadratic functions as transformations of the function $f(x) = x^2$.
- c. Investigate and explain characteristics of quadratic functions, including domain, range, vertex, axis of symmetry, zeros, intercepts, extrema, intervals of increase and decrease, and rates of change.

Student Activities Overview and Answer Key

Station 1

Given equations in the form $y = (x - h)^2$, where h is an integer, students will graph a series of parabolas, finding the y -intercept and the axis of symmetry. Students will explore the relationship between the value of a and the position of the parabola with respect to the x - and y -axes. Students should also begin to understand the relationship between the equation of the parabola and the axis of symmetry.

Answers

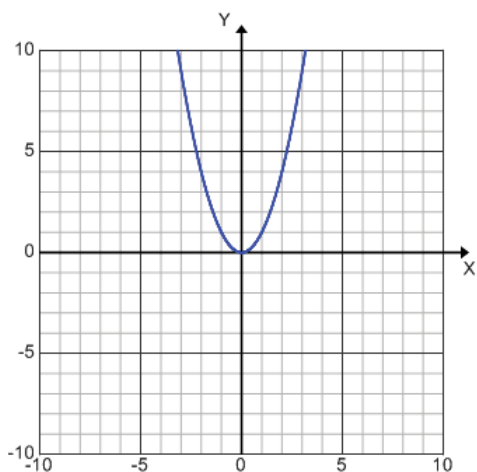
1.

x	y
0	0
1	1
2	4
3	9
-1	1
-2	4
-3	9

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Station Activities Set 1: Quadratic Transformations

Instruction

2.



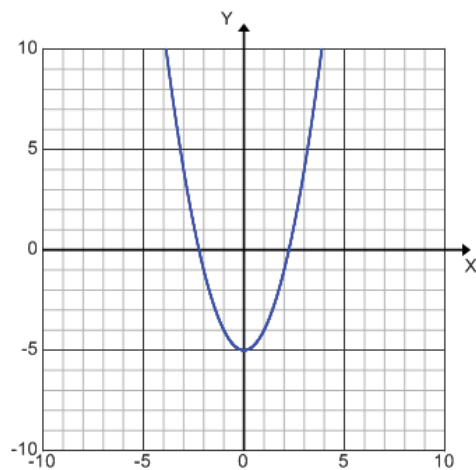
3. (0, 0)

4. (0, 2)

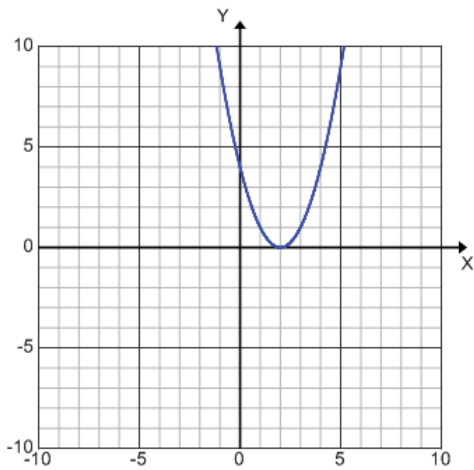
5. $y = x^2 + 2$

6. $y = x^2 - 5$

7.



8.



9. $x = 2$

10. $x = -3$

Station 2

Given equations in the form $y = ax^2$, students graph parabolas. Students will compare graphs to explore the relationship between the coefficient of x and the width of the parabola.

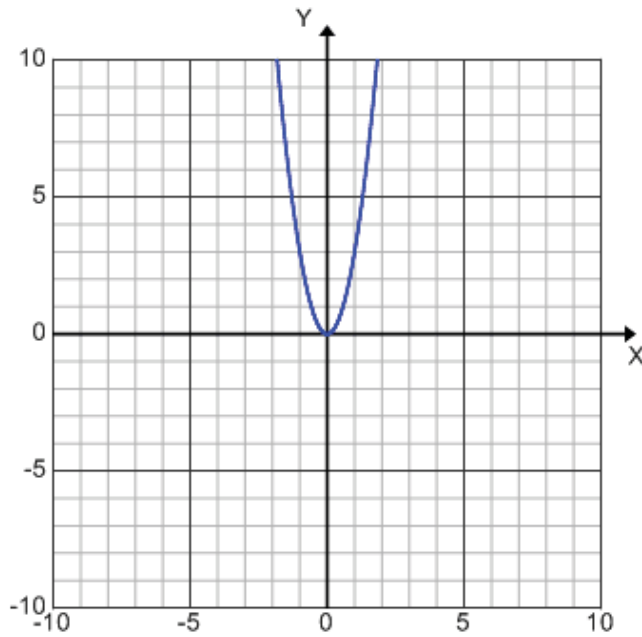
Answers

1.

x	y
0	0
1	3
2	12
3	27
-1	3
-2	12
-3	27

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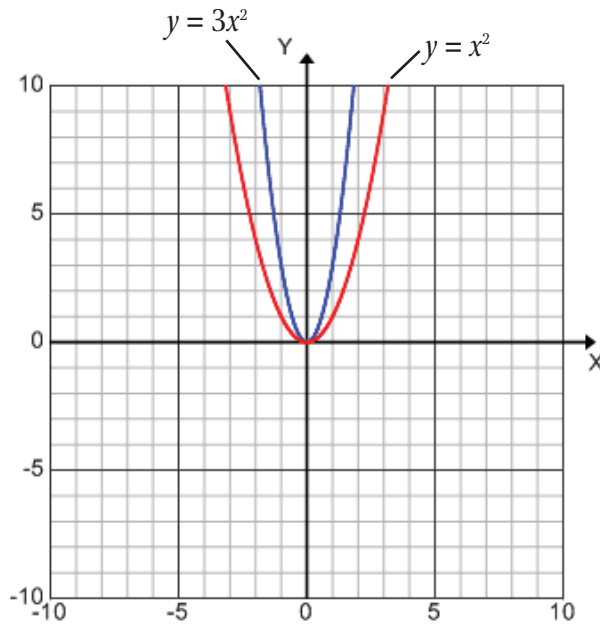
Instruction



2. (0, 0)

3. $x = 0$

4.

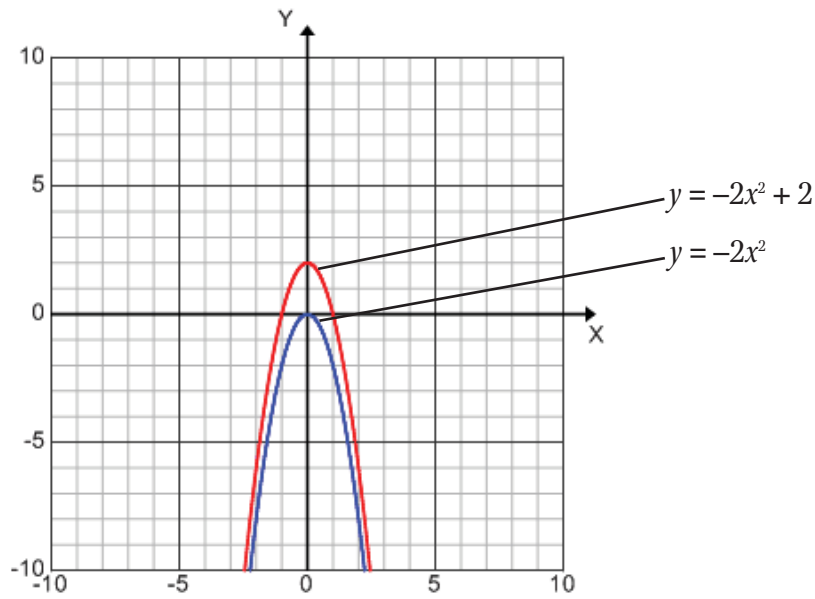


The parabola $3x^2$ is narrower than the parabola x^2 .

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Instruction

5–6.



7. $(0, 2)$
8. The parabola moves vertically.
9. The parabola changes in width.

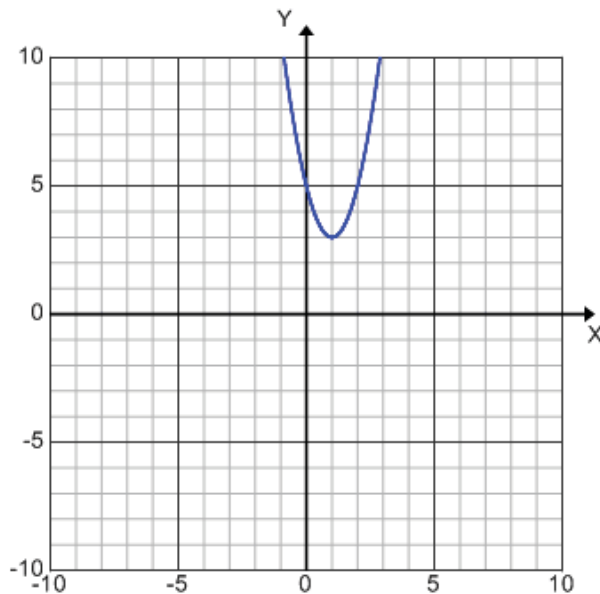
Station 3

Given equations in the form $y = (x + h)^2 - k$, students graph parabolas. Students will find the y -intercept and the axis of symmetry from both the graph and the equation, and will begin working towards an understanding of the vertex of a parabola.

Answers

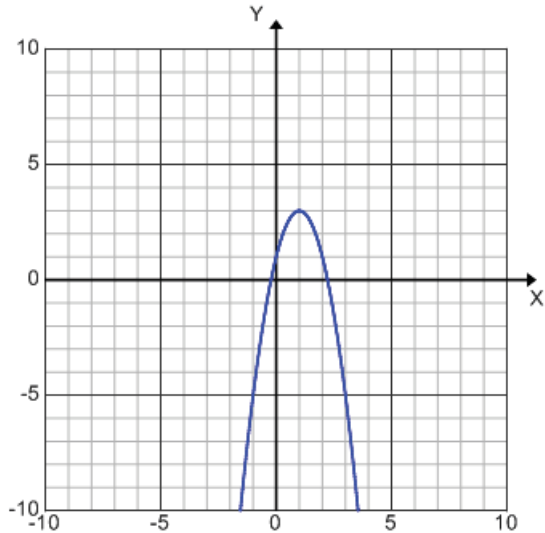
1.

x	y
0	5
1	3
2	5
3	11
4	21
-1	11
-2	21
-3	35



- 2. $x = 1$
- 3. $(0, 5)$

4. The parabola would open downward.



5. (0, 3)

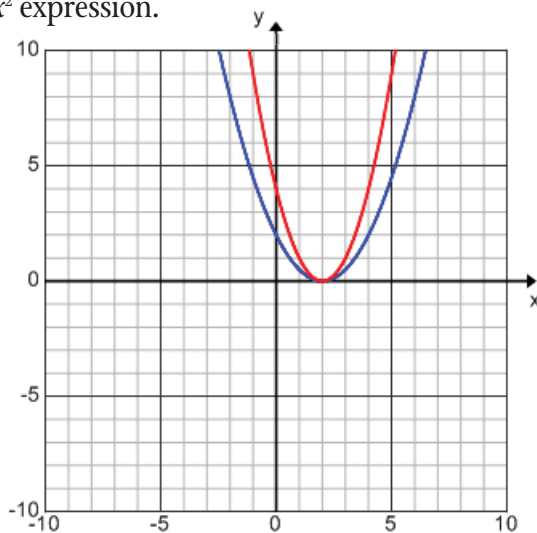
$$y = \frac{1}{2}(x - 2)^2 + 1$$

$$y = \frac{1}{2}(0 - 2)^2 + 1$$

$$y = \frac{1}{2}(4) + 1$$

$$y = 3$$

6. It will be wider, because the higher the coefficient of the x^2 expression, the narrower the parabola. The coefficient of the second x expression is 1, which is higher than $\frac{1}{2}$, the coefficient of the first x^2 expression.



7. $(0, -17)$

8. $y = 3(x - 2)^2 + 10$

The new y -intercept is $(0, 22)$, so solve for the value of a .

$$y = 3(x - 2)^2 + a$$

$$22 = 3(0 - 2)^2 + a$$

$$22 = 3(4) + a$$

$$22 = 12 + a$$

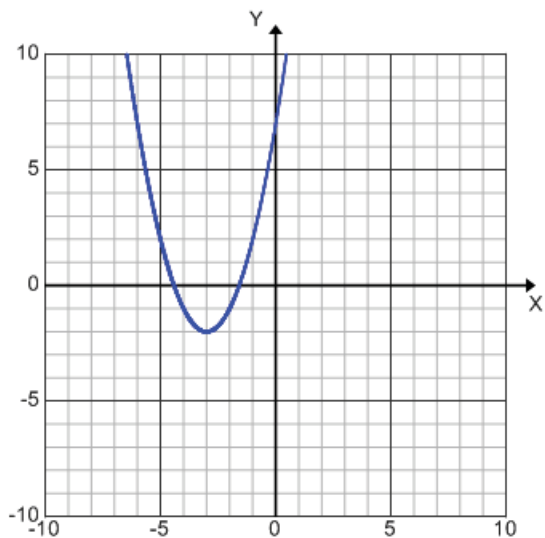
$$10 = a$$

Station 4

Students use two methods (completing the square and finding the midpoint of the x -intercepts) to convert the equations of parabolas from quadratic form to vertex form. Students will graph to check their work and to understand the correlation between the different forms and the graph. Students should recognize that a parabola's axis of symmetry always runs through its vertex. Students should also understand the relationship between the coordinates of the vertex and the equation in vertex form.

Answers

1.



2. $x = -3$

3. $y = x^2 + 6x + 7$

$$y = x^2 + 6x + 7 + 2 - 2$$

$$y = (x^2 + 6x + 9) - 2$$

$$y = (x + 3)^2 - 2$$

4. $(-3, -2)$

5. $(4, 1)$

6.

$$y = \frac{x^2}{2} - 4x + 9$$

$$y = \frac{x^2}{2} - 4x + 8 + 1$$

$$y = \frac{1}{2}(x^2 - 8x + 16) + 1$$

$$y = \frac{1}{2}(x - 4)^2 + 1$$

7. $(0, 9)$

8. $x = 4$

9. Yes. The parabola opens out from the vertex. The vertex contains the only y -coordinate that is not repeated in the range.

10. Because the parabola is symmetrical, the axis of symmetry will intersect the midpoint of the line between the x -intercepts. The midpoint is at $(0, 0)$. That means the x -coordinate at the vertex must be 0, because the axis of symmetry intersects the vertex. If $x = 0, y = -4$, so the coordinates of the vertex are $(0, -4)$.

Materials List/Set Up

Station 1 student activity sheet, graph paper

Station 2 student activity sheet, colored pencils or pens, graph paper

Station 3 student activity sheet, colored pencils or pens, graph paper

Station 4 student activity sheet, graph paper

Discussion Guide

To support students in reflecting on the activities and to gather some formative information about student learning, use the following prompts to facilitate a class discussion to “debrief” the station activities.

Prompts/Questions

1. What is a function’s axis of symmetry? Does every parabola have one?
2. What is a y -intercept?
3. How do you find the coordinates of a parabola’s y -intercept?
4. Compare the equations $y = a(x + h)^2 - k$ and $y = ax^2 + 2axh + ah^2 - k$. Do you think they express the same thing? How could you find out?

Think, Pair, Share

Have students jot down their own responses to questions, then discuss with a partner (who was not in their station group), and then discuss as a whole class.

Suggested Appropriate Responses

1. An axis of symmetry is the line that divides the graph of the function into two symmetrical halves. Every parabola has an axis of symmetry.
2. A y -intercept is the point at which a function crosses the y -axis.
3. Set x equal to 0 and solve the function for y .
4. Students should multiply out the equation in vertex form to find the equation in quadratic form.

Possible Misunderstandings/Mistakes

- Incorrectly calculating the value of y -coordinates from x -coordinates
- Incorrectly graphing parabolas, either from incorrect calculations or from a misunderstanding of graphing itself
- Not understanding the definition of the vertex
- Assuming that the vertex is unrelated to the axis of symmetry
- Incorrectly factoring quadratic equations

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- Making simple arithmetical errors in completing the square
- Not understanding the arithmetical manipulations involved in completing the square
- Confusing the y -intercept with the x -intercept
- Confusing the vertex coordinates h and k
- English language learners may struggle with the questions that ask for written explanations. Encourage these students to write out the numeric operations involved and then describe their work out loud.

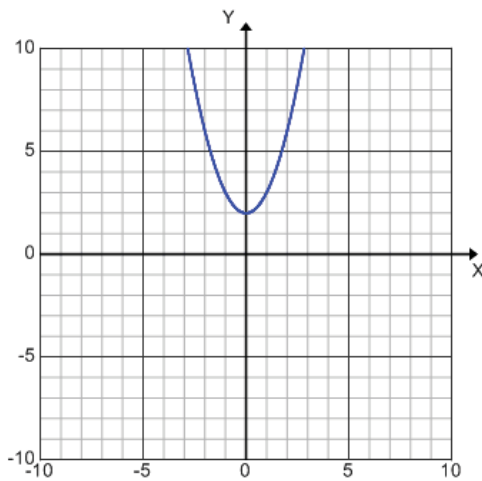
UNIT • QUADRATICS AND COMPLEX NUMBERS
Station Activities Set 1: Quadratic Transformations**Station 1**

Work as a group to answer the questions. Construct graphs without the aid of a graphing calculator. Show all your work and label the axes of each graph.

1. Given the parabola $y = x^2$, complete the table below with the y coordinates for the following values of x .

x	y
0	
1	
2	
3	
-1	
-2	
-3	

2. Use the coordinates from your table to graph the parabola on graph paper.
3. What are the coordinates for the parabola's y -intercept?
4. Look at the parabola below. What is its y -intercept?

**continued**

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Station Activities Set 1: Quadratic Transformations**Station 2**

Work with your group to explore the relationship between a quadratic function and its graph.

1. Given the equation $y = 3x^2$, complete the table with the values of y and graph the parabola.

x	y
0	
1	
2	
3	
-1	
-2	
-3	

2. What are the coordinates of this parabola's y -intercept?
3. What is the equation of its axis of symmetry?
4. On the graph from problem 1, draw the parabola $y = x^2$ in a contrasting color. In words, compare the two parabolas.
5. Graph the parabola $y = -2x^2$. Complete the table if you need a reference.

x	y
0	
1	
2	
3	
-1	
-2	
-3	

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6. On the same graph, in a contrasting color, graph the parabola $y = -2x^2 + 2$. Label each parabola.

7. What are the coordinates of the y -intercept of $y = -2x^2 + 2$?

8. What happens to the graph of a parabola when you add a numeric constant to its equation, as in problem 6?

9. What happens to the graph of a parabola when the x^2 expression is given a numeric coefficient, as in problems 1 and 5? (*Hint*: Compare the parabola $y = 3x^2$ to $y = x^2$.)

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Work with your group to answer the questions below.

1. Complete the table for the parabola $y = 2(x - 1)^2 + 3$. Graph the parabola on graph paper.

x	y
0	
1	
2	
3	
4	
-1	
-2	
-3	

2. What is the equation for this parabola's axis of symmetry?
3. What are the coordinates of this parabola's y -intercept?
4. How would this graph change if the parabola's equation changed to $y = -2(x - 1)^2 + 3$? Graph the new parabola to check your answer.
5. What are the coordinates of the y -intercept of the parabola $y = \frac{1}{2}(x - 2)^2 + 1$?

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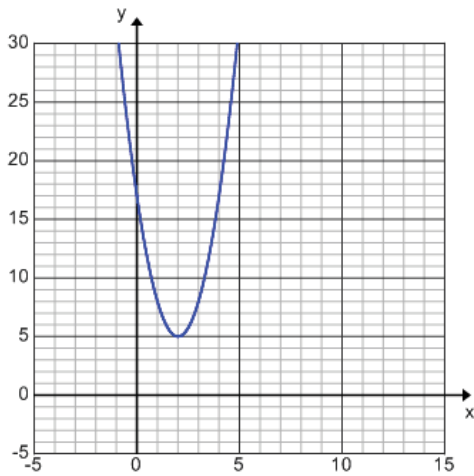
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6. Do you think that the graph of $y = \frac{1}{2}(x - 2)^2$ will be wider or narrower than the graph of $y = (x - 2)^2$? Why? Graph both parabolas, in contrasting colors, to check your answer.

7. Look at the graph below. The equation for this parabola is $y = 3(x - 2)^2 + 5$. What is its y -intercept?



8. How would you write the equation for a similar parabola with a y -intercept 5 units higher? Show your work. Write out an explanation in words if necessary.

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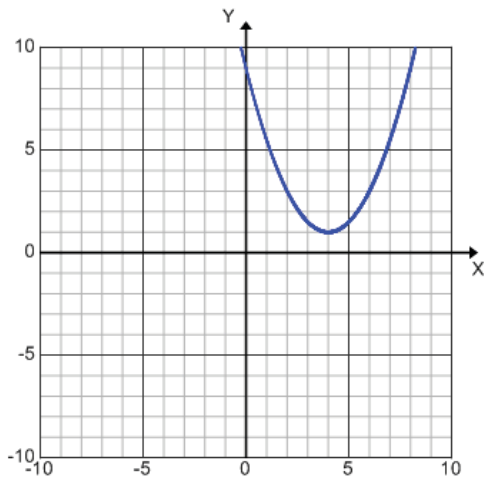
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Station 4

Work with your group to answer the questions below.

1. Graph the parabola $y = x^2 + 6x + 7$ on graph paper.
2. Give the equation for its axis of symmetry.
3. *Optional:* Complete the square to give the equation for the parabola in vertex form. Show your work.
4. What are the coordinates of the vertex of this parabola?
5. Look at the graph below. What are the coordinates of the vertex of this parabola?



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6. The equation for this parabola is $y = \frac{x^2}{2} - 4x + 9$. Find the vertex in order to convert the equation to vertex form. Show your work.
7. What are the coordinates of the y -intercept?
8. What is the equation for the axis of symmetry?
9. Does a parabola's axis of symmetry always run through its vertex? Why or why not?
10. Look at the graph below, which shows the parabola $y = x^2 - 4$. The coordinates of the parabola's x -intercepts are $(2, 0)$ and $(-2, 0)$. How could you use this information to find the coordinates of the parabola's vertex? Explain, showing your work.

