Alignment Analysis of the Walch Education Analytic Geometry Program to the Common Core Georgia Performance Standards for Analytic Geometry

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This study was conducted for Walch Education, Portland, ME.

Table of Contents

I. Introduction 1
II. Methodology 2
III. Findings 4
Strength of Alignment 4
Depth of Knowledge 5
Conclusion 5
References 6
Appendix A: Qualifications of the Researcher
Appendix B: Alignment Ratings
Appendix C: Mathematics Depth of Knowledge Definitions

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I. Introduction

In a standards based education system, alignment between expectations for student learning, instruction, and assessment is critical. Alignment expert, Dr. Norman Webb, defines alignment as is the degree to which the various components of an educational system—expectations, curricula, instruction, and assessments—are in agreement and work together to achieve desired goals for student achievement. Close alignment helps educators focus on the desired content and ensures that students have a fair opportunity to learn and to demonstrate their knowledge and understanding. (Webb, 1997, 2005)

The Analytic Geometry program is a comprehensive set of instructional materials developed by Walch Education specifically to address the Common Core Georgia Performance Standards (CCGPS) for Analytic Geometry. The complete package of instructional materials includes the following components:

- eight resource books intended for teachers (one for each of seven units and a program overview);
- resource books for students; and
- digital enhancements: digital resources for teachers, digital warm-up activities, and digital instruction.

To ensure proper alignment of the instructional program to the CCGPS, Walch Education selected Amy S. Burkam, president of Lothlorien Consulting, to conduct an independent alignment study. A summary of her qualifications and experience is provided in Appendix A.

II. Methodology

The purpose of this study is to address one key question.

> To what degree does the Walch Education Analytic Geometry program provide instructional materials that address the content specified by each CCGPS for Analytic Geometry?

The criteria used in this study are adapted from the work of Dr. Norman Webb (Webb, 1997, 2005). The Webb methodology was developed to examine the alignment between assessments and standards. Webb describes four alignment criteria: Categorical Concurrence, Depth of Knowledge, Range of Knowledge, and Balance of Representation. Webb's benchmarks for meeting each criterion, and in some cases, the criterion itself, are based on the premise that assessments typically survey the content specified by the standards. For this study, Webb's criteria are adapted to serve the assertion that instruction must provide sufficient opportunities for students to master all content and skills specified by the standards.

Categorical concurrence is the degree to which standards and assessments incorporate the same content. For assessments, the categorical concurrence criterion is evaluated by determining whether the assessment includes items measuring some content from each standard. To meet the criterion for depth-of-knowledge (DoK) consistency, the cognitive processes required to answer the assessment tasks must be as demanding as the expectations defined by the standards. Typically, the DoK criterion is met for an assessment if at least 50% of the items corresponding to a standard are at or above the DoK level assigned to the performance indicator. The range-of-knowledge criterion is used to judge whether the span of knowledge defined by a standard is comparable to the span of knowledge required to correctly answer the assessment items. Fifty percent of the objectives for a standard must have at least one related assessment item to meet this alignment criterion. Webb's range-of-knowledge criterion only considers the number of objectives within a standard assessed; it does not consider how the assessment items are distributed among the objectives. The balance-of-representation criterion is used to indicate the degree to which one objective is given more emphasis on the assessment than another.

To evaluate the alignment of the Walch Education Analytic Geometry materials to the CCGPS for Analytic Geometry, this study focuses on the categorical concurrence and range-of-knowledge alignment criteria. To be considered aligned, instructional materials must provide opportunities for students to learn the material associated with every standard and every objective within the standard. The balance-of-representation criterion does not apply to the evaluation of instructional materials. If every standard and objective is thoroughly covered, any variation in emphasis is assumed to be an intentional artifact of the standards. This study does not include a formal evaluation of the cognitive demand required to complete the exercises included in the instructional materials. However, the overall rigor of exercises was noted while conducting the study and general observations are provided in the results section of this report.

To conduct the study, the researcher compared each CCGPS for Analytic Geometry to the content of the teacher and student resource materials to determine the degree to which the lessons provide opportunities for students to learn, practice, and apply the full range of knowledge and skills specified by each standard. In instances where the CCGPS are further elaborated by indicators, the content of the resource materials was also evaluated against each indicator. The definitions listed in Table 1 were applied to each CCGPS.

Table 1
Definitions for Evaluating Strength of Alignment

Code	Description
S (Strong)	The lessons and student resources fully address the content specified by the standard (or indicators, when present). The lessons provide sufficient opportunities for students to learn, practice, and apply the full range of knowledge and skills specified by the standard.
P (Partial)	The lessons and student resources address the content specified by the standard/indicator superficially, or cover less sophisticated skills or content than represented by the standard/indicator, or cover only a portion of the specified skills or content.
N (No Relationship)	The lessons do not address the content of the standard/indicator.

III. Findings

Strength of Alignment

Appendix B contains tables showing the alignment relationship of each CCGPS to the Walch Education Analytic Geometry Program. Table 2 summarizes the findings.

Table 2

Strength of Alignment

Unit	*n*	Strength of Alignment					
		Strong		Partial		No Relationship	
		n	%	*n*	%	*n*	%
1. Similarity, Congruence, and Proofs							
Standards	13	13	100%	0	0%	0	0%
Indicators	2	2	100%				
2. Right Triangle Trigonometry							
Standards	3	3	100%	0	0%	0	0%
Indicators	0						
3. Circles and Volume							
Standards	9	9	100%	0	0%	0	0%
Indicators	0						
4. Extending the Number System							
Standards	7	7	100%	0	0%	0	0%
Indicators	0						
5. Quadratic Functions							
Standards	19	19	100%	0	0%	0	0%
Indicators	11	11	100%	0	0%	0	0%
6. Modeling Geometry							
Standards	4	4	100%	0	0%	0	0%
Indicators	0			0	0%	0	0%
7. Applications of Probability							
Standards	7	7	100%	0	0%	0	0%
Indicators	0			0	0%	0	0%

The Walch Education Analytic Geometry instructional materials demonstrate strong alignment relationships to all standards and indicators (100%) specified by the CCGPS for Analytic Geometry. The lessons provide ample opportunities for students to learn, practice, and apply the full range of knowledge and skills specified by each standard and indicator. The Student Resource Books and Digital Enhancements correspond directly to the lessons in the Teacher Resource and reflect the same level of alignment.

Depth of Knowledge

Although this study does not include a formal evaluation of the cognitive processes required of students to complete the exercises provided with the lessons, the researcher made several noteworthy observations regarding the nature and rigor of the exercises.

1. The exercises and assessment items include a variety of formats or item types.
2. Problems incorporate an assortment of real-world contexts.
3. Exercises span the range of cognitive complexity, with emphasis on application and higher order thinking skills.

The exercises include variety of formats, such as:

- multiple-choice;
- short answer (e.g., graph the equation or solve the problem and show your work);
- extended constructed-response, requiring the application of skills and written explanations of the mathematical processes; and
- performance tasks, requiring hands-on engagement and exploration of mathematical concepts.

Webb defines four depth-of-knowledge levels for mathematics, with Level One being the lowest and Level Four the highest. Definitions for each level are provided in Appendix. C. In particular, the constructed response and performance activities require cognitive processes at Levels Three and Four. Students are expected to delve deeply into the content, make connections, and solve complex problems.

Conclusion

The Walch Education Analytic Geometry instructional materials demonstrate strong alignment relationships to the content specified by the CCGPS for Analytic Geometry. All content specified by the standards is addressed in a manner consistent with the breadth and depth indicated by the standards. The lessons provide sufficient opportunities for students to learn, practice, and apply the full range of knowledge and skills specified by each standard and indicator. The Student Resource Book and Digital Enhancements correspond directly to the lessons in the Teacher Resource and reflect the same level of alignment. The Walch Education Analytic Geometry program fully meets the categorical concurrence and range-of-knowledge alignment criteria and, based on these criteria, is considered completely and precisely aligned to the CCGPS.

References

Webb, N.L. (1997). Criteria for alignment of expectations and assessments in mathematics and science education. (NISE Research and Monograph No. 8). Madison: University of Wisconsin—Madison, National Institute for Science Education. Washington DC: Council of Chief State School Officers.

Webb, N.L. (2005). *Web Alignment Tool (WAT) Training Manual Draft Version 1.1.* Madison: University of Wisconsin, Wisconsin Center for Education Research.

Appendix A

Qualifications of the Researcher

Appendix A: Qualifications of the Researcher

Assessment specialist, Amy S. Burkam, president of Lothlorien Consulting, LLC, conducted this study. A former high school science teacher, Ms. Burkam has worked in large scale assessment since 1985 when she joined the staff at Measured Progress, formerly Advanced Systems in Measurement and Evaluation, Inc. During her 22 year tenure with Measured Progress, she worked as a test developer for K-12 mathematics and science, managed and directed multiple statewide assessment programs, and directed the Curriculum and Assessment division. In her role as Curriculum and Assessment director, Ms. Burkam supervised and coordinated all aspects of item and test development. In 2007, Ms. Burkam established a private consulting firm to provide assessment-related services to state departments of education, assessment companies, districts, and schools. Services include providing management and leadership for item and test development initiatives; designing and conducting alignment studies; working with educators to develop content standards, item banks, assessments, and performance standards; facilitating meetings; and reviewing and editing K-12 assessment items.

Since 2007, Ms. Burkam designed and conducted the following alignment studies and predictive analyses (crosswalks).

- Crosswalks between WIN Learning's Career Readiness Objectives and the Common Core State Standards for Mathematics and English Language Arts
- Crosswalks between the Common Core State Standards for Mathematics and English Language Arts and the standards measured by the ERB Comprehensive Testing Program
- Alignment Analysis of the Vermont Alternate Assessment Portfolio (VTAAP) for Reading, Mathematics, and Science (March 2011)
- Alignment Analysis of Maine's 2009-2010 Personalized Alternate Assessment Portfolio (PAAP) Alternate Grade Level Expectations for Reading, Mathematics, and Science
- Alignment Analysis of the New England Common Assessment Program Expectations for Reading and Mathematics and a Form of the SAT Test of Reasoning
- Alignment Analysis of the Maine Science *Learning Results* and the Science Portion of Maine's 2009 Comprehensive Assessment System for Grades 5, 8, and 11
- Alignment Analysis of the Maine High School Mathematics *Learning Results* and a Form of the May 2009 SAT Mathematics Assessment
- Alignment Analysis of the Maine High School Reading *Learning Results* and a Form of the May 2009 SAT Reading Assessment

- Alignment Analysis of the Maine High School Mathematics *Learning Results* and a Form of the May 2008 SAT Mathematics
- Crosswalks between the Massachusetts and Tennessee reading and mathematics content standards to predict the degree of alignment between a Massachusetts item bank and the Tennessee standards
- Crosswalks between the Maine *Learning Results* for mathematics, reading, and science and the New England Common Assessment Program (NECAP) Grade Level Expectations to predict the degree of alignment between the NECAP assessment and Maine's *Learning Results*

Appendix B

Alignment Ratings

Appendix B: Alignment Ratings

Unit	Georgia Performance Standard	Lesson(s)	Teacher Resource Page #	Strength of Alignment
1	MCC9-12.G.SRT.1 Verify experimentally the properties of dilations given by a center and a scale factor.	1.1.1-2	U-6-42	S
	a. A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged.	1.1.1	U-6-28	S
	b. The dilation of a line segment is longer or shorter in the ratio given by the scale factor.	1.1.2	U-29-42	S
	MCC9-12.G.SRT.2 Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.	1.6.1	U-324-346	S
	MCC9-12.G.SRT.3 Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar. Prove theorems involving similarity	1.6.2	U-347-362	S
	MCC9-12.G.SRT.4 Prove theorems about triangles. Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.	1.7.1-3	U-374-432	S
	MCC9-12.G.SRT.5 Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures. Understand congruence in terms of rigid motions	1.7.4	U1-435-455	S
	MCC9-12.G.CO.6 Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.	1.4.1-2	U1-216-261	S
	MCC9-12.G.CO.7Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.	1.5.1	U1-276-294	S
	MCC9-12.G.CO.8 Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.	1.5.2	U1-295-315	S

Unit	Georgia Performance Standard	Lesson(s)	Teacher Resource Page #	Strength of Alignment
1	MCC9-12.G.CO.9 Prove theorems about lines and angles. Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.	1.8.1-2	U1-468-524	S
	MCC9-12.G.CO.10 Prove theorems about triangles. Theorems include: measures of interior angles of a triangle sum to 180 degrees; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.	1.9.1-4	U1-539-658	S
	MCC9-12.G.CO.11 Prove theorems about parallelograms. Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals. Make geometric constructions	1.10-1-2	U1-668-729	S
	MCC9-12.G.CO.12 Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.	1.2.1-3	U1-54-128	S
	MCC9-12.G.CO.13Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle	1.3.1-3	U1-137-204	S

Unit	Georgia Performance Standard	Lesson(s)	Teacher Resource Page #	Strength of Alignment
2	MCC9-12.G.SRT.6 Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.	2.1.1	U2-6-33	S
	MCC9-12.G.SRT.7 Explain and use the relationship between the sine and cosine of complementary angles.	2.1.2	U2-34-50	S
	MCC9-12.G.SRT.8 Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.	2.2.1-3	U2-59-122	S
3	MCC9-12.G.C.1 Prove that all circles are similar.	3.1.1	U3-5-27	S
	MCC9-12.G.C.2 Identify and describe relationships among inscribed angles, radii, and chords. Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.	3.1.1-3	U3-5-62	S
	MCC9-12.G.C.3 Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.	3.2.1-3	U3-71-127	S
	MCC9-12.G.C.4 (+)Construct a tangent line from a point outside a given circle to the circle. Find arc lengths and areas of sectors of circles	3.3.1	U3-136-161	S
	MCC9-12.G.C.5 Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector. Explain volume formulas and use them to solve problems	3.4.1-2	U3-169-193	S
	MCC9-12.G.GMD.1 Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. Use dissection arguments, Cavalieri's principle, and informal limit arguments.	3.5.1-2	U3-200-243	S
	MCC9-12.G.GMD.2 (+) Give an informal argument using Cavalieri's principle for the formulas for the volume of a sphere and other solid figures.	3.5.3	U3-244-262	S
	MCC9-12.G.GMD.3 Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.★	3.5.2-3	U3-222-262	S
	MCC8.G.9 Know the formulas for the volume of cones, cylinders, and spheres and use them to solve real-world and mathematical problems. (transition standard)	3.5.2-3	U3-222-262	S

Unit	Georgia Performance Standard	Lesson(s)	Teacher Resource Page #	Strength of Alignment
4	MCC9-12.N.RN.1 Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents.	4.1.1	U4-4-17	S
	MCC9-12.N.RN.2 Rewrite expressions involving radicals and rational exponents using the properties of exponents. Use properties of rational and irrational numbers.	4.1.1-2	U4-4-30	S
	MCC9-12.N.RN.3 Explain why the sum or product of rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational. Perform arithmetic operations with complex numbers.	4.1.2	U4-18-30	S
	MCC9-12.N.CN.1 Know there is a complex number i such that $i^2 = -1$, and every complex number has the form $a + bi$ with a and b real.	4.3.1	U4-67-78	S
	MCC9-12.N.CN.2 Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.	4.3.2-3	U4-79-103	S
	MCC9-12.N.CN.3 (+) Find the conjugate of a complex number; use conjugates to find ~~moduli and~~ quotients of complex numbers. Perform arithmetic operations on polynomials	4.3.4	U4-104-118	S
	MCC9-12.A.APR.1 Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials. (Focus on polynomial expressions that simplify to forms that are linear or quadratic in a positive integer power of x)	4.2.1-2	U4-35-61	S

Unit	Georgia Performance Standard	Lesson(s)	Teacher Resource Page #	Strength of Alignment
5	MCC9-12.N.CN.7 Solve quadratic equations with real coefficients that have complex solutions. Interpret the structure of expressions	5.2.1 5.2.3-5	U5-36-47 U5-63-103	S
	MCC9-12.A.SSE.1 Interpret expressions that represent a quantity in terms of its context.★ (Focus on quadratic functions; compare with linear and exponential functions studied in Coordinate Algebra.)	5.1.1-2	U5-4-29	S
	MCC9-12.A.SSE.1a Interpret parts of an expression, such as terms, factors, and coefficients.★ (Focus on quadratic functions; compare with linear and exponential functions studied in Coordinate Algebra.)	5.1.1	U5-4-16	S
	MCC9-12.A.SSE.1b Interpret complicated expressions by viewing one or more of their parts as a single entity.★ (Focus on quadratic functions; compare with linear and exponential functions studied in Coordinate Algebra.)	5.1.2	U5-17-29	S
	MCC9-12.A.SSE.2 Use the structure of an expression to identify ways to rewrite it. (Focus on quadratic functions; compare with linear and exponential functions studied in Coordinate Algebra.)Write expressions in equivalent forms to solve problems	5.2.2-3 5.2.5	U5-48-75 U5-89-103	S
	MCC9-12.A.SSE.3 Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.★(Focus on quadratic functions; compare with linear and exponential functions studied in Coordinate Algebra.)	5.3.1-3	U5-109-167	S
	MCC9-12.A.SSE.3a Factor a quadratic expression to reveal the zeros of the function it defines.★	5.3.1-2	U5-109-153	S
	MCC9-12.A.SSE.3b Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines. ★Create equations that describe numbers or relationships	5.3.3	U5-154-167	S
	MCC9-12.A.CED.1 Create equations and inequalities in one variable and use them to solve problems. Include equations arising from ~~linear and~~ quadratic functions, ~~and simple rational and exponential functions.~~★	5.2.1-5	U5-36-103	S
	MCC9-12.A.CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.★ (Focus on quadratic functions; compare with linear and exponential functions studied in Coordinate Algebra.)	5.3.1-3	U5-109-167	S

Unit	Georgia Performance Standard	Lesson(s)	Teacher Resource Page #	Strength of Alignment
5	MCC9-12.A.CED.4 Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. (Focus on quadratic functions; compare with linear and exponential functions studied in Coordinate Algebra.) Solve equations and inequalities in one variable	5.3.4	U5-168-181	S
	MCC9-12.A.REI.4 Solve quadratic equations in one variable.	5.2.1-5	U5-36-103	S
	MCC9-12.A.REI.4a Use the method of completing the square to transform any quadratic equation in *x* into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.	5.2.3-4	U5-63-88	S
	MCC9-12.A.REI.4b Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers *a* and *b.* Solve systems of equations	5.2.1-5	U5-36-103	S
	MCC9-12.A.REI.7 Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. Interpret functions that arise in applications in terms of the context	5.4.1-2	U5-189-221	S
	MCC9-12.F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; ~~and periodicity~~.★	5.5.1	U5-230-250	S
	MCC9-12.F.IF.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.★ (Focus on quadratic functions; compare with linear and exponential functions studied in Coordinate Algebra.)	5.5.2	U5-251-264	S
	MCC9-12.F.IF.6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.★ (Focus on quadratic functions; compare with linear and exponential functions studied in Coordinate Algebra.)Analyze functions using different	5.5.3	U5-265-279	S

Unit	Georgia Performance Standard	Lesson(s)	Teacher Resource Page #	Strength of Alignment
	representations			
5	MCC9-12.F.IF.7 Graph functions expressed symbolically and show key features of The graph, by hand in simple cases and using technology for more complicated cases. ★ (Focus on quadratic functions; compare with linear and exponential functions studied in Coordinate Algebra.)	5.6.1	U5-287-306	S
	MCC9-12.F.IF.7aGraph ~~linear and~~ quadratic functions and show intercepts, maxima, and minima.★	5.6.1	U5-287-306	S
	MCC9-12.F.IF.8 Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. (Focus on quadratic functions; compare with linear and exponential functions studied in Coordinate Algebra.)	5.6.2	U5-307-326	S
	MCC9-12.F.IF.8a Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.	5.6.2	U5-307-326	S
	MCC9-12.F.IF.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). (Focus on quadratic functions; compare with linear and exponential functions studied in Coordinate Algebra.)Build a function that models a relationship between two quantities	5.6.3	U5-327-346	S
	MCC9-12.F.BF.1 Write a function that describes a relationship between two quantities.★ (Focus on quadratic functions; compare with linear and exponential functions studied in Coordinate Algebra.)	5.7.1-2	U5-352-386	S
	MCC9-12.F.BF.1a Determine an explicit expression, a recursive process, or steps for calculation from a context. (Focus on quadratic functions; compare with linear and exponential functions studied in Coordinate Algebra.)	5.7.1	U5-352-371	S
	MCC9-12.F.BF.1b Combine standard function types using arithmetic operations. (Focus on quadratic functions; compare with linear and exponential functions studied in Coordinate Algebra.)	5.7.2	U5-372-386	S

Unit	Georgia Performance Standard	Lesson(s)	Teacher Resource Page #	Strength of Alignment
5	MCC9-12.F.BF.3 Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. (Focus on quadratic functions; compare with linear and exponential functions studied in Coordinate Algebra.)Construct and compare linear, quadratic, and exponential models and solve problems	5.8.1-2	U5-392-439	S
	MCC9-12.F.LE.3 Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.★	5.6.3	U5-327-346	S
	MCC9-12.S.ID.6 Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.★	5.9.1-2	U5-446-497	S
	MCC9-12.S.ID.6a Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize ~~linear,~~ quadratic, ~~and exponential~~ models.★	5.9.1-2	U5-446-497	S
6	MCC9-12.A.REI.7 Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. Translate between the geometric description and the equation for a conic section	6.3.1	U6-100-130	S
	MCC9-12.G.GPE.1Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.	6.1.1	U6-5-30	S
	MCC9-12.G.GPE.2Derive the equation of a parabola given a focus and directrix. Use coordinates to prove simple geometric theorems algebraically	6.1.2	U6-31-58	S
	MCC9-12.G.GPE.4Use coordinates to prove simple geometric theorems algebraically. (Restrict to context of circles and parabolas)	6.2.1	U6-66-92	S

Unit	Georgia Performance Standard	Lesson(s)	Teacher Resource Page #	Strength of Alignment
7	MCC9-12.S.CP.1 Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events (“or,” “and,” “not”).★	7.1.1	U7-6-32	S
	MCC9-12.S.CP.2 Understand that two events *A* and *B* are independent if the probability of *A* and *B* occurring together is the product of their probabilities, and use this characterization to determine if they are independent.★	7.1.3	U7-48-68	S
	MCC9-12.S.CP.3 Understand the conditional probability of *A* given *B* as *P(A and B)/P(B)*, and interpret independence of *A* and *B* as saying that the conditional probability of *A* given *B* is the same as the probability of *A*, and the conditional probability of *B* given *A* is the same as the probability of *B*.★	7.2.1	U7-78-101	S
	MCC9-12.S.CP.4 Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities.★	7.2.2	U7-102-124	S
	MCC9-12.S.CP.5 Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. ★Use the rules of probability to compute probabilities of compound events in a uniform probability model	7.2.1-2	U7-78-124	S
	MCC9-12.S.CP.6 Find the conditional probability of *A* given *B* as the fraction of *B*’s outcomes that also belong to *A*, and interpret the answer in terms of the model.★	7.2.1-2	U7-78-124	S
	MCC9-12.S.CP.7Apply the Addition Rule, *P(A or B) = P(A) +P(B) – P(A and B)*, and interpret the answer in terms of the model.★	7.1.2	U7-33-47	S

Appendix C

Depth of Knowledge Definitions for Mathematics

Appendix C: Mathematics Depth of Knowledge Definitions

(Dr. Norman Webb)

Level 1 (Recall) includes the recall of information such as a fact, definition, term, or a simple procedure, as well as performing a simple algorithm or applying a formula. A one-step, well-defined, algorithmic procedure is considered a Level 1 activity. Keywords that signify Level 1 exercises include "identify," "recall," "recognize," "use," and "measure." Verbs such as "describe" and "explain" could be classified at different levels, depending on what the student must describe and explain.

Level 2 (Skill/Concept) involves the engagement of some mental processing beyond a habitual response. A Level 2 assessment item requires students to decide how to approach the problem or activity, whereas a Level 1 item requires students to provide a rote response, perform a memorized algorithm, follow a set procedure (like a recipe), or perform a clearly defined series of steps. Keywords or phrases that generally distinguish a Level 2 item include "classify," "organize," "estimate," "make observations," "collect and display data," and "compare data." These actions imply more than one step. For example, to compare data a student must identify characteristics of the objects or phenomenon and then group or order the objects. Verbs such as "explain," "describe," or "interpret," could be classified at different levels depending on the object of the action. Interpreting information from a simple graph is a Level 2 task. Interpreting information from a complex graph that requires determining which features of the graph need to be considered or how information from the graph can be aggregated is at Level 3. Level 2 activities are not limited to number skills, but can involve visualization skills and probability skills. Other Level 2 activities include noticing and describing non-trivial patterns, explaining the purpose and use of experimental procedures; carrying out experimental procedures; making observations and collecting data; classifying, organizing, and comparing data; and organizing and displaying data in tables, graphs, and charts.

Level 3 (Strategic Thinking) requires reasoning, planning, using evidence, and a higher level of thinking than the previous two levels. In most instances, exercises that require students to explain their thinking are at Level 3. Activities that require students to make conjectures are also at this level. The cognitive demands at Level 3 are complex and abstract. The complexity does not result from the fact that there are multiple answers, a possibility for both Levels 1 and 2, but because the task requires more demanding reasoning. An activity that has more than one possible answer *and* requires students to justify their responses would most likely be at Level 3. Other Level 3 activities include drawing conclusions from observations; citing evidence and developing a logical argument for concepts; explaining phenomena in terms of concepts; and using concepts to solve problems.

Level 4 (Extended Thinking) requires complex reasoning, planning, developing, and thinking, most likely over an extended period of time. The extended time period is not a distinguishing factor if the work is repetitive and does not require significant conceptual understanding and higher-order thinking. For example, an activity that involves measuring the water temperature of a river each day for a month and then constructing a graph would be classified at Level 2. However, if the student must conduct a river study that requires consideration of several variables, the task is likely at Level 4. At Level 4, the cognitive demands of the task should be high and the work should be complex. Students should be required to make several connections—relate ideas *within* the content area or *among* content areas—and select one approach among many alternatives. Level 4 activities include developing and proving conjectures; designing and conducting experiments; making connections between a finding and related concepts and phenomena; combining and synthesizing ideas into new concepts; and critiquing experimental designs.