THE YOU CAN DO IT! GUIDE TO Algebra

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General Problems

1. Combining Positive and Negative Numbers

- Every number has a sign, making it either positive or negative. A plus (+) sign may be implied. For example, the number 5 is really positive 5 or +5. The number –1 is negative 1.
- A number can have many plus and/or minus signs preceding it.
- **To tell if a number is positive or negative, count the number of minus signs before a number.** An even (2,4,6,8, etc.) total of minus signs before a number makes the number positive, regardless of the amount of plus signs before it. An odd total (1,3,5,7,9, etc.) of minus signs makes the number negative.

  Examples:

  – – – – 14 (four minus signs make it positive 14.)

  + – + – 6 (two minus signs make it positive 6.)

  + + + – 1 (one minus sign makes it negative 1.)

  + – – – – – – 42 (six minus signs makes it positive 42.)
- Parentheses don't affect the counting of minus signs before a number. For example:

  – (– 4) (two minus signs makes it positive 4.)

Step 1: First, identify each number's sign.

Examples:

– (– 6) + – 2	(two minus signs before 6 make it positive 6.) (one minus sign before 2 makes it negative 2.)
1 – – + 3	(zero minus signs before 1 make it positive 1.) (two minus signs before 3 make it positive 3.)

Step 2: Then combine the numbers.

To combine two negative numbers: Ignore their signs and add them. Then attach a negative sign to the final answer.

Example: Combine –2 and –4. Ignore the signs. 2 + 4 = 6. Attach a minus. Answer: –6

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To combine one positive and one negative number: Ignore their signs and subtract the lesser number from the greater number. Attach the original sign of the greater number to the final answer.

Example: Combine –8 and 4. Ignore the signs. 8 – 4 = 4. Attach the original sign of the 8 (a minus sign). Answer: –4.

To combine two positive numbers: Add them.

Problems, 1. Combining Positive and Negative Numbers

Combine the following numbers:

1. 3 – – 2

2. 10 + – – – 5

3. – 4 + – – 2

4. + + – – 2 – – + + – – 1

5. + – 6 + – – –1

2. Moving Terms Across the Equal Sign

- Moving terms means taking a term (number or variable term) and moving it to the opposite side of the equal sign in order to "isolate the variable."

- Every term is either positive (it has a plus sign in front of it, or the plus is implied, e.g., the number 100), or negative (it has a minus sign in front of it).

- When moving a term to the other side, change its sign.

  Examples:

  To move the +2 to the right side of the equal sign, change it to a –2.

  $$3 + 2 = 5$$
  $$3 = 5 - 2$$

  To move the –2 to the right side of the equal sign, change it to a +2.

  $$x - 2 = 5$$
  $$x = 5 + 2$$

  To move the x to the right side of the equal sign, make it a $-x$.

  $$x + y = 2$$
  $$y = 2 - x$$

  To move the $-4y$ to the right side of the equal sign, make it a $+ 4y$.

  $$x - 4y = 12$$
  $$x = 12 + 4y$$

  Move all terms to the right side.

  $x + 3a - 3b = 2$
  $x + 3a = 2 + 3b$ (moved the $-3b$ over)
  $x = 2 + 3b - 3a$ (moved the $3a$ over)
  $0 = 2 + 3b - 3a - x$ (moved the x over)

- Moving terms is a shortcut for adding something to both sides, or subtracting something from both sides.

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Problems, 2. Moving Terms Across the Equal Sign

Move the number to the right-hand side of the equation:

1. $x + 2 = 5$

2. $x - 12 = 14$

3. $x - 2 = 6$

4. $x - 2 = 0$

5. $2x + 4y + 1 = 5$

46. Doing Function Problems as "Function Machines"

A function is like a machine. In a gum ball machine, you put something (a coin) in, and you get something (a gum ball) out. In a function machine, you put a number in and you get the answer to a problem out.

- The function machine looks like this: F (X) = $2x + 3$

  The F (X) on the left of "=" tells you the machine's name. It's called "F-machine".

  F (X) — (X) here means "machine." **It's *not* a variable.**

  "F" tells the letter name of the machine.

  The letter "F" and the word "machine" form the full name "F-machine."

  F (X) = $2x + 3$
  only means: "F-machine" is $2x + 3$.

  "is" means "="

  Example: G (X) = $3x + 7$ . . . Means G-machine is $3x + 7$

  Example: H (X) = $2x + 9$ . . . Means H-machine is $2x + 9$

- A full problem might look like this:
  If G (X) = $7x + 10$, find G (4).

  This means: If G-machine is $7x + 10$, what do you get when you put 4 into the machine everywhere the machine has a variable x?

  G-machine is $7x + 10$
  Put 4 in: 7 (4) + 10
  which equals: 28 + 10
  which equals: 38
  When you put a 4 into this G-machine, you get 38 out.

  If R (X) = $7x + 10$, find R (2).

  This means: If R-machine is $7x + 10$ and you put a 2 into the variable, you get
  7 (2) + 10
  14 + 10
  = 24

  Note: When you put the number into the machine, put ( ) around the number.

  If R (X) = $x^2 + x$, find R (3).

  This means: If R-machine is $x^2 + x$, when 3 is put into the machine for **all** the x variables, you get

  $(3)^2 + (3)$
  9 + 3
  = 12

  In addition to meaning "machine" the (X) in R (X) = $x^2 + x$ tells you that you're plugging in for the x variable. For a function machine like A (X) = $xy + 3y$, the (X) tells you to plug in for the x variable only.

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Problems, 46. Doing Function Problems as "Function Machines"

Use the function-machine method to solve the following:

1. If F (X) = $5x + 1$, find F (2).

2. If F (X) = $2x + 3$, find F (8).

3. If G (X) = $x + 1$, find G (4).

4. If H (X) = $x^2 + x + 1$, find H (2).

5. If S (X) = $x^2 - 2$, find S (–2).

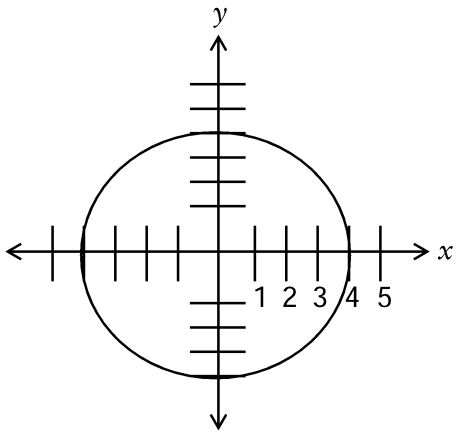
54. Circles

The standard form of a circle with a center at (0,0) is:

$$x^2 + y^2 = (\text{radius})^2$$

Example: $x^2 + y^2 = 16$
$x^2 + y^2 = (4)^2$ (because $16 = 4 \times 4$)
The radius is 4.

Put points four units left, right, up, and down from the center (0,0) and make a circle.



But sometimes the center is not at (0,0). Then the equation standard form is:

$$(x - h)^2 + (y - k)^2 = (\text{radius})^2$$

Example: $(x - 3)^2 + (y+2)^2 = 4$

Step 1. Take each parentheses separately, say it is equal to 0, and solve.

$x - 3 = 0$ $\qquad$ $y + 2 = 0$
$x = 3$ $\qquad$ $y = -2$

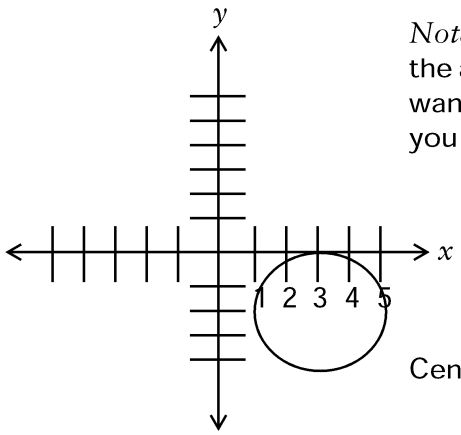
- The center is at (3, –2).

Step 2. Find the radius by getting the square root of the number on the right side of the equation.

The radius is the square root of 4, which is 2.

- Go two left, right, up, and down from the center point (3, –2), and put points. Connect the points to form a circle.

(continued)



Note: The center point is **not** part of the actual circle graph. So you may not want to put the center point in when you graph circles for class.

Center point is (3, –2).

You may also encounter problems requiring completing the square in two places.

Example: $x^2 + 4x + y^2 + 6y = 3$

Complete the squares: $(x + \mathbf{2})^2 + (y + \mathbf{3})^2 = 3 + 4 + 9$

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Problems, 54. Circles

Find the center of the circle, the radius of the circle, and graph:

1. $x^2 + y^2 = 4$

2. $(x - 2)^2 + y^2 = 25$

3. $(x - 1)^2 + (y - 2)^2 = 9$

4. $(x + 4)^2 + (y - 5)^2 = 16$

5. $x^2 + 2x + y^2 + 6x = 6$