Alignment Analysis of the *Common Core State Standards* *Integrated Pathway: Mathematics II* to the Utah Core State Standards for Mathematics II

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This study was conducted for Walch Education, Portland, ME.

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I. Introduction

In a standards based education system, alignment between expectations for student learning, instruction, and assessment is critical. Alignment expert, Dr. Norman Webb, defines alignment as is the degree to which the various components of an educational system—expectations, curricula, instruction, and assessments—are in agreement and work together to achieve desired goals for student achievement. Close alignment helps educators focus on the desired content and ensures that students have a fair opportunity to learn and to demonstrate their knowledge and understanding. (Webb, 1997, 2005)

The *Common Core State Standards Integrated Pathway: Mathematics II* is a comprehensive set of instructional materials developed by Walch Education specifically to address the second year of the integrated pathway for high school mathematics outlined in Appendix A of the Common Core State Standards. The complete package of instructional materials includes the following components.

- seven resource books intended for teachers (one for each of six units and a program overview);
- two resource books for students (units one through four and units five through six);
- digital enhancements: digital resources for teachers, digital warm-up activities, and digital instruction

To ensure proper alignment of the instructional program to Utah's Core State Standards, Walch Education selected Amy S. Burkam, president of Lothlorien Consulting, to conduct an independent alignment study. A summary of her qualifications and experience is provided in Appendix A.

II. Methodology

The purpose of this study is to address one key question.

> To what degree does the *Common Core State Standards Integrated Pathway: Mathematics II* provide instructional materials that address the content specified by each Utah core standard for Secondary Mathematics II?

The criteria used in this study are adapted from the work of Dr. Norman Webb (Webb, 1997, 2005). The Webb methodology was developed to examine the alignment between assessments and standards. Webb describes four alignment criteria: Categorical Concurrence, Depth of Knowledge, Range of Knowledge, and Balance of Representation. Webb's benchmarks for meeting each criterion, and in some cases, the criterion itself, are based on the premise that assessments typically survey the content specified by the standards. For this study, Webb's criteria are adapted to serve the assertion that instruction must provide sufficient opportunities for students to master all content and skills specified by the standards.

Categorical concurrence is the degree to which standards and assessments incorporate the same content. For assessments, the categorical concurrence criterion is evaluated by determining whether the assessment includes items measuring some content from each standard. To meet the criterion for depth-of-knowledge (DoK) consistency, the cognitive processes required to answer the assessment tasks must be as demanding as the expectations defined by the standards. Typically, the DoK criterion is met for an assessment if at least 50% of the items corresponding to a standard are at or above the DoK level assigned to the performance indicator. The range-of-knowledge criterion is used to judge whether the span of knowledge defined by a standard is comparable to the span of knowledge required to correctly answer the assessment items. Fifty percent of the objectives for a standard must have at least one related assessment item to meet this alignment criterion. Webb's range-of-knowledge criterion only considers the number of objectives within a standard assessed; it does not consider how the assessment items are distributed among the objectives. The balance-of-representation criterion is used to indicate the degree to which one objective is given more emphasis on the assessment than another.

To evaluate the alignment of the *Common Core State Standards Integrated Pathway: Mathematics II* to the Utah Core State Standards, this study focuses on the categorical concurrence and range-of-knowledge alignment criteria. To be considered aligned, instructional materials must provide opportunities for students to learn the material associated with every standard and every objective within the standard. The balance-of-representation criterion does not apply to the evaluation of instructional materials. If every standard and objective is thoroughly covered, any variation in emphasis is assumed to be an intentional artifact of the standards. This study does not include a formal evaluation of the cognitive demand required to complete the exercises included in the instructional materials. However, the overall rigor of exercises was noted while conducting the study and general observations are provided in the results section of this report.

To conduct the study, the researcher compared each Utah core standard to the content of the teacher and student resource materials to determine the degree to which the lessons provide opportunities for students to learn, practice, and apply the full range of knowledge and skills specified by each standard. The definitions listed in Table 1 were applied to each Utah standard.

Table 1
Definitions for Evaluating Strength of Alignment

Code	Description
S (Strong)	The lessons and student resources fully address the content specified by the standard (or indicators below the standard, when present). The lessons provide sufficient opportunities for students to learn, practice, and apply the full range of knowledge and skills specified by each standard.
P (Partial)	The lessons and student resources address the content specified by the standard/indicator superficially, or cover less sophisticated skills or content than represented by the standard/indicator, or cover only a portion of the specified skills or content.
N (No Relationship)	The lessons do not address the content of the standard/indicator.

III. Findings

Strength of Alignment

Appendix B contains tables showing the alignment relationship of each Utah standard and indicator to the content presented in the *Common Core State Standards Integrated Pathway: Mathematics II* instructional materials. Table 2 summarizes the findings.

Table 2
Strength of Alignment

Unit	*n*	Strength of Alignment					
		Strong		Partial		No Relationship	
		n	%	*n*	%	*n*	%
1. Extending the Number System							
Standards	6	6	100%	0	0%	0	0%
Indicators	0						
2. Quadratic Functions and Modeling							
Standards	10	10	100%	0	0%	0	0%
Indicators	7	7	100%	0	0%	0	0%
3. Expressions and Equations							
Standards	11	11	100%	0	0%	0	0%
Indicators	7	7	100%	0	0%	0	0%
4. Applications of Probability							
Standards	11	11	100%	0	0%	0	0%
Indicators	0						
5. Similarity, Right Triangle Trigonometry, and Proof							
Standards	13	13	100%	0	0%	0	0%
Indicators	2	2	100%	0	0%	0	0%
6. Circles with and without Coordinates							
Standards	10	10	100%	0	0%	0	0%
Indicators	0						

Table II shows that the *Common Core State Standards Integrated Pathway: Mathematics II* instructional materials demonstrate strong alignment relationships to all standards and indicators (100%) specified by the Utah Core State Standards for Secondary Mathematics II.

One concept, periodicity, was missing from the lessons that address standard F.IF.4.

> F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts;*

intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.★

The instructional notes presented in the Utah Core Standards document state, “Focus on quadratic functions; compare with linear and exponential functions studied in Secondary Mathematics I” (Utah State Office of Education, 2012). In light of this statement, the alignment relationship of the instructional materials to standard F.IF.4 was considered “strong,” and the omission was regarded as intentional and appropriate.

The *Common Core State Standards Integrated Pathway: Mathematics II* instructional materials thoroughly satisfy the Utah Core Standards for Mathematics II. The lessons provide ample opportunities for students to learn, practice, and apply the full range of knowledge and skills specified by each standard and indicator. The Student Resource Books and Digital Enhancements correspond directly to the lessons in the Teacher Resource Books and reflect the same level of alignment.

Depth of Knowledge

Although this study does not include a formal evaluation of the cognitive processes required of students to complete the exercises provided with the lessons, the researcher made several noteworthy observations regarding the nature and rigor of the exercises.

1. The exercises and assessment items include a variety of formats or item types.
2. Problems incorporate an assortment of real-world contexts.
3. Exercises span the range of cognitive complexity, with emphasis on application and higher order thinking skills.

The exercises include variety of formats, such as:

- multiple-choice;
- short answer (e.g., graph the equation or solve the problem and show your work);
- extended constructed-response, requiring the application of skills, written justification of answers, and explanation of mathematical processes; and
- performance tasks (stations), requiring hands-on engagement and exploration of mathematical concepts.

Webb defines four depth-of-knowledge levels for mathematics, with Level One being the lowest and Level Four the highest. Definitions for each level are provided in Appendix. C. In particular, the constructed response and performance activities require cognitive processes at Levels Three and Four. Students are expected to delve deeply into the content, make connections, and solve complex problems.

Conclusion

The *Common Core State Standards Integrated Pathway: Mathematics II* instructional materials demonstrate strong alignment relationships to the content specified by the Utah Core State Standards for Secondary Mathematics II. All content specified by the standards is addressed in a manner consistent with the breadth and depth indicated by the standards. The lessons provide sufficient opportunities for students to learn, practice, and apply the full range of knowledge and skills specified by each standard and indicator. The Student Resource Book and Digital Enhancements correspond directly to the lessons in the Teacher Resource and reflect the same level of alignment. The *Common Core State Standards Integrated Pathway: Mathematics II* fully meets the categorical concurrence and range-of-knowledge alignment criteria and, based on these criteria, is considered completely and precisely aligned to the Utah Core State Standards.

References

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Appendix A

Qualifications of the Researcher

Appendix A: Qualifications of the Researcher

Assessment specialist, Amy S. Burkam, president of Lothlorien Consulting, LLC, conducted this study. A former high school science teacher, Ms. Burkam has worked in large scale assessment since 1985 when she joined the staff at Measured Progress, formerly Advanced Systems in Measurement and Evaluation, Inc. During her 22 year tenure with Measured Progress, she worked as a test developer for K-12 mathematics and science, managed and directed multiple statewide assessment programs, and directed the Curriculum and Assessment division. In her role as Curriculum and Assessment director, Ms. Burkam supervised and coordinated all aspects of item and test development. In 2007, Ms. Burkam established a private consulting firm to provide assessment-related services to state departments of education, assessment companies, districts, and schools. Services include providing management and leadership for item and test development initiatives; designing and conducting alignment studies; working with educators to develop content standards, item banks, assessments, and performance standards; facilitating meetings; and reviewing and editing K-12 assessment items.

Since 2007, Ms. Burkam designed and conducted the following alignment studies and predictive analyses (crosswalks).

- Crosswalks between WIN Learning's Career Readiness Objectives and the Common Core State Standards for Mathematics and English Language Arts
- Crosswalks between the Common Core State Standards for Mathematics and English Language Arts and the standards measured by the ERB Comprehensive Testing Program
- Alignment Analysis of the Vermont Alternate Assessment Portfolio (VTAAP) for Reading, Mathematics, and Science (March 2011)
- Alignment Analysis of Maine's 2009-2010 Personalized Alternate Assessment Portfolio (PAAP) Alternate Grade Level Expectations for Reading, Mathematics, and Science
- Alignment Analysis of the New England Common Assessment Program Expectations for Reading and Mathematics and a Form of the SAT Test of Reasoning
- Alignment Analysis of the Maine Science *Learning Results* and the Science Portion of Maine's 2009 Comprehensive Assessment System for Grades 5, 8, and 11
- Alignment Analysis of the Maine High School Mathematics *Learning Results* and a Form of the May 2009 SAT Mathematics Assessment
- Alignment Analysis of the Maine High School Reading *Learning Results* and a Form of the May 2009 SAT Reading Assessment

- Alignment Analysis of the Maine High School Mathematics *Learning Results* and a Form of the May 2008 SAT Mathematics
- Crosswalks between the Massachusetts and Tennessee reading and mathematics content standards to predict the degree of alignment between a Massachusetts item bank and the Tennessee standards
- Crosswalks between the Maine *Learning Results* for mathematics, reading, and science and the New England Common Assessment Program (NECAP) Grade Level Expectations to predict the degree of alignment between the NECAP assessment and Maine's *Learning Results*

Appendix B

Alignment Ratings

Appendix B: Alignment Ratings

Unit	Utah Core State Standard	Lesson(s)	Teacher Resource Page #	Strength of Alignment
1	N.RN.1 Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. *For example, we define $5^{1/3}$ to be the cube root of 5 because we want $(5^{1/3})^3 = 5^{(1/3)3}$ to hold, so $(5^{1/3})^3$ must equal 5.*	1.1.1	U1-5-17	S
	N.RN.2 Rewrite expressions involving radicals and rational exponents using the properties of exponents.	1.1.1 1.1.2	U1-5-30	S
	N.RN.3 Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.	1.1.2	U1-18-30	S
	N.CN.1 Know there is a complex number i such that $i^2 = -1$, and every complex number has the form $a + bi$ with a and b real.	1.3.1	U1-67-78	S
	N.CN.2 Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.	1.3.2 1.3.3	U1-79-103	S
	A.APR.1 Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.	1.2.1 1.2.2	U1-35-61	S

Unit	Utah Core State Standard	Lesson(s)	Teacher Resource Page #	Strength of Alignment
2	F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.* ★	2.2.1	U2-56-77	S
	F.IF.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*★	2.2.2	U2-78-91	S
	F.IF.6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.★	2.2.3	U2-92-106	S
	F.IF.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★	2.1.1-2 2.4.1-3	U2-4-49 U2-156-244	S
	F.IF.7.a Graph linear and quadratic functions and show intercepts, maxima, and minima.	2.1.1 2.1.2	U2-4-49	S
	F.IF.7.b Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.	2.4.1 2.4.2 2.4.3	U2-156-244	S
	F.IF.8 Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.	2.1.2	U2-28-49	S
	F.IF.8.a Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.	2.1.2 2.5.1	U2-28-49 U2-255-268	S

Unit	Utah Core State Standard	Lesson(s)	Teacher Resource Page #	Strength of Alignment
2	F.IF.8.b Use the properties of exponents to interpret expressions for exponential functions. *For example, identify percent rate of change in functions such as* $y = (1.02)^t$, $y = (0.97)^t$, $y = (1.01)^{12t}$, $y = (1.2)^{t/10}$, *and classify them as representing exponential growth or decay.*	2.5.1	U2-255-268	S
	F.IF.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.*	2.5.2	U2-269-287	S
	F.BF.1 Write a function that describes a relationship between two quantities.	2.3.1 2.3.2	U2-114-133 U2-134-148	S
	F.BF.1.a Determine an explicit expression, a recursive process, or steps for calculation from a context.	2.3.1	U2-114-133	S
	F.BF.1.b Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.*	2.3.2	U2-134-148	S
	F.BF.3 Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.	2.6.1 2.6.2	U2-296-339	S
	F.BF.4 Find inverse functions.	2.7.1	U2-348-363	S
	F.BF.4.a Solve an equation of the form $f(x) = c$ for a simple function f that has an inverse and write an expression for the inverse. *For example,* $f(x) = 2x3$ *or* $f(x) = (x+1)/(x-1)$ *for* $x \neq 1$.	2.7.1	U2-348-363	S
	F.LE.3 Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.	2.5.2	U2-269-287	S

Unit	Utah Core State Standard	Lesson(s)	Teacher Resource Page #	Strength of Alignment
3	A.SSE.1 Interpret expressions that represent a quantity in terms of its context.	3.1.1 3.1.2	U3-4-29	S
	A.SSE.1.a Interpret parts of an expression, such as terms, factors, and coefficients.	3.1.1	U3-4-16	S
	A.SSE.1.b Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret $P(1+r)^n$ as the product of P and a factor not depending on P.*	3.1.2	U3-17-29	S
	A.SSE.2 Use the structure of an expression to identify ways to rewrite it. For example, see $x^4 – y^4$ *as* $(x^2)^2 – (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 – y^2)(x^2 + y^2)$.	3.2.2-3 3.2.5	U3-47-75 U3-90-104	S
	A.SSE.3 Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.★	3.3.1-3 3.6.1	U3-111-169 U3-351-374	S
	A.SSE.3.a Factor a quadratic expression to reveal the zeros of the function it defines.	3.3.1 3.3.2	U3-111-155	S
	A.SSE.3.b Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.	3.3.3	U3-156-169	S
	A.SSE.3.c Use the properties of exponents to transform expressions for exponential functions. *For example the expression 1.15^t can be rewritten as $(1.15^{1/12})^{12t} \approx 1.012^{12t}$ to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.*	3.6.1	U3-351-374	S
	A.CED.1 Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.	3.2.1-5 3.5.1 3.5.3	U3-36-104 U3-247-268 U3-302-341	S
	A.CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.	3.3.1-3 3.5.2	U3-111-169 U3-269-301	S

Unit	Utah Core State Standard	Lesson(s)	Teacher Resource Page #	Strength of Alignment
3	A.CED.4 Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. For example, rearrange Ohm's law V = IR to highlight resistance R.	3.3.4	U3-170-183	S
	A.REI.4 Solve quadratic equations in one variable.	3.2.1-5	U3-36-104	S
	A.REI.4.a Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.	3.2.3 3.2.4	U3-63-89	S
	A.REI.4.b Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.	3.2.1-5 3.4.2	U3-36-104 U3-212-237	S
	N.CN.7 Solve quadratic equations with real coefficients that have complex solutions.	3.4.2	U3-212-237	S
	N.CN.8 (+) Extend polynomial identities to the complex numbers. *For example, rewrite* $x^2 + 4$ *as* $(x + 2i)(x - 2i)$.	3.4.1	U3-190-211	S
	N.CN.9 (+) Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.	3.4.2	U3-212-237	S
	A.REI.7 Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. *For example, find the points of intersection between the line* $y = -3x$ *and the circle* $x^2 + y^2 = 3$.	3.7.1-2	U3-382-420	S

Unit	Utah Core State Standard	Lesson(s)	Teacher Resource Page #	Strength of Alignment
4	S.CP.1 Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events (“or,” “and,” “not”).	4.1.1	U4-6-32	S
	S.CP.2 Understand that two events *A* and *B* are independent if the probability of *A* and *B* occurring together is the product of their probabilities, and use this characterization to determine if they are independent.	4.1.3	U4-48-68	S
	S.CP.3 Understand the conditional probability of *A* given *B* as *P(A and B)/P(B)*, and interpret independence of A and B as saying that the conditional probability of *A* given *B* is the same as the probability of *A,* and the conditional probability of *B* given *A* is the same as the probability of *B*.	4.2.1	U4-79-102	S
	S.CP.4 Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. *For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results.*	4.2.2	U4-103-125	S
	S.CP.5 Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. *For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.*	4.2.1-2	U4-79-125	S
	S.CP.6 Find the conditional probability of *A* given *B* as the fraction of *B*’s outcomes that also belong to *A*, and interpret the answer in terms of the model.	4.2.1-2	U4-79-125	S
	S.CP.7 Apply the Addition Rule, *P(A or B) = P(A) + P(B) – P(A and B)*, and interpret the answer in terms of the model.	4.1.2	U4-33-46	S

Unit	Utah Core State Standard	Lesson(s)	Teacher Resource Page #	Strength of Alignment
4	S.CP.8 (+) Apply the general Multiplication Rule in a uniform probability model, *P(A and B) = P(A)P(B|A) = P(B)P(A|B)*, and interpret the answer in terms of the model.	4.2.3	U4-126-147	S
	S.CP.9 (+) Use permutations and combinations to compute probabilities of compound events and solve problems.	4.3.1-2	U4-155-190	S
	S.MD.6 (+) Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator).	4.4.1	U4-198-217	S
	S.MD.7 (+) Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game).	4.4.2	U4-218-237	S

Unit	Utah Core State Standard	Lesson(s)	Teacher Resource Page #	Strength of Alignment
5	G.SRT.1 Verify experimentally the properties of dilations given by a center and a scale factor:	5.2.1-2	U5-34-71	S
	G.SRT.1.a A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged.	5.2.1	U5-34-46	S
	G.SRT.1.b The dilation of a line segment is longer or shorter in the ratio given by the scale factor.	5.2.2	U5-57-71	S
	G.SRT.2 Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.	5.3.1	U5-82-104	S
	G.SRT.3 Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.	5.3.2	U5-105-120	S
	G.CO.9 Prove theorems about lines and angles. *Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints*.	5.5.1-2	U5-226-282	S
	G.CO.10 Prove theorems about triangles. *Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.*	5.6.1-4	U5-297-416	S
	G.CO.11 Prove theorems about parallelograms. *Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals*.	5.7.1-2	U5-426-487	S
	G.SRT.4 Prove theorems about triangles. *Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.*	5.4.1-3	U5-132-190	S

Unit	Utah Core State Standard	Lesson(s)	Teacher Resource Page #	Strength of Alignment
5	G.SRT.5 Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.	5.4.4	U5-191-213	S
	G.GPE.6 Find the point on a directed line segment between two given points that partitions the segment in a given ratio.	5.1.1	U5-3-25	S
	G.SRT.6 Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.	5.8.1	U5-496-525	S
	G.SRT.7 Explain and use the relationship between the sine and cosine of complementary angles.	5.8.2	U5-524-540	S
	G.SRT.8 Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.★	5.9.1-3	U5-550-614	S
	F.TF.8 Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to calculate trigonometric ratios.	5.9.4	U5-615-631	S

Unit	Utah Core State Standard	Lesson(s)	Teacher Resource Page #	Strength of Alignment
6	G.C.1 Prove that all circles are similar.	6.1.1	U6-5-27	S
	G.C.2 Identify and describe relationships among inscribed angles, radii, and chords. *Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.*	6.1.1-3	U6-5-62	S
	G.C.3 Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.	6.2.1-3	U6-71-127	S
	G.C.4 (+) Construct a tangent line from a point outside a given circle to the circle.	6.3.1	U6-136-161	S
	G.C.5 Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.	6.4.1-2	U6-169-193	S
	G.GPE.1 Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.	6.6.1	U6-252-277	S
	G.GPE.2 Derive the equation of a parabola given a focus and directrix.	6.6.2	U6-278-305	S
	G.GPE.4 Use coordinates to prove simple geometric theorems algebraically. *For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1, $\sqrt{3}$) lies on the circle centered at the origin and containing the point (0, 2).*	6.7.1	U6-313-339	S
	G.GMD.1 Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. *Use dissection arguments, Cavalieri's principle, and informal limit arguments.*	6.5.1-2	U6-200-244	S
	G.GMD.3 Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.★	6.5.2	U6-222-244	S

Appendix C

Depth of Knowledge Definitions for Mathematics

Appendix C: Mathematics Depth of Knowledge Definitions

(Dr. Norman Webb)

Level 1 (Recall) includes the recall of information such as a fact, definition, term, or a simple procedure, as well as performing a simple algorithm or applying a formula. A one-step, well-defined, algorithmic procedure is considered a Level 1 activity. Keywords that signify Level 1 exercises include “identify,” “recall,” “recognize,” “use,” and “measure.” Verbs such as “describe” and “explain” could be classified at different levels, depending on what the student must describe and explain.

Level 2 (Skill/Concept) involves the engagement of some mental processing beyond a habitual response. A Level 2 assessment item requires students to decide how to approach the problem or activity, whereas a Level 1 item requires students to provide a rote response, perform a memorized algorithm, follow a set procedure (like a recipe), or perform a clearly defined series of steps. Keywords or phrases that generally distinguish a Level 2 item include “classify,” “organize,” ”estimate,” “make observations,” “collect and display data,” and “compare data.” These actions imply more than one step. For example, to compare data a student must identify characteristics of the objects or phenomenon and then group or order the objects. Verbs such as “explain,” “describe,” or “interpret,” could be classified at different levels depending on the object of the action. Interpreting information from a simple graph is a Level 2 task. Interpreting information from a complex graph that requires determining which features of the graph need to be considered or how information from the graph can be aggregated is at Level 3. Level 2 activities are not limited to number skills, but can involve visualization skills and probability skills. Other Level 2 activities include noticing and describing non-trivial patterns, explaining the purpose and use of experimental procedures; carrying out experimental procedures; making observations and collecting data; classifying, organizing, and comparing data; and organizing and displaying data in tables, graphs, and charts.

Level 3 (Strategic Thinking) requires reasoning, planning, using evidence, and a higher level of thinking than the previous two levels. In most instances, exercises that require students to explain their thinking are at Level 3. Activities that require students to make conjectures are also at this level. The cognitive demands at Level 3 are complex and abstract. The complexity does not result from the fact that there are multiple answers, a possibility for both Levels 1 and 2, but because the task requires more demanding reasoning. An activity that has more than one possible answer *and* requires students to justify their responses would most likely be at Level 3. Other Level 3 activities include drawing conclusions from observations; citing evidence and developing a logical argument for concepts; explaining phenomena in terms of concepts; and using concepts to solve problems.

Level 4 (Extended Thinking) requires complex reasoning, planning, developing, and thinking, most likely over an extended period of time. The extended time period is not a distinguishing factor if the work is repetitive and does not require significant conceptual understanding and higher-order thinking. For example, an activity that involves measuring the water temperature of a river each day for a month and then constructing a graph would be classified at Level 2. However, if the student must conduct a river study that requires consideration of several variables, the task is likely at Level 4. At Level 4, the cognitive demands of the task should be high and the work should be complex. Students should be required to make several connections—relate ideas *within* the content area or *among* content areas—and select one approach among many alternatives. Level 4 activities include developing and proving conjectures; designing and conducting experiments; making connections between a finding and related concepts and phenomena; combining and synthesizing ideas into new concepts; and critiquing experimental designs.