Alignment Analysis of the
Common Core State Standards
Integrated Pathway: Mathematics III
to the Utah Core Standards for Secondary Mathematics III

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I. Introduction

In a standards based education system, alignment between expectations for student learning, instruction, and assessment is critical. Alignment expert, Dr. Norman Webb, defines alignment as is the degree to which the various components of an educational system—expectations, curricula, instruction, and assessments—are in agreement and work together to achieve desired goals for student achievement. Close alignment helps educators focus on the desired content and ensures that students have a fair opportunity to learn and to demonstrate their knowledge and understanding. (Webb, 1997, 2005)

The *Common Core State Standards Integrated Pathway: Mathematics III* is a comprehensive set of instructional materials developed by Walch Education specifically to address the third year of the integrated pathway for high school mathematics outlined in Appendix A of the Common Core State Standards.

To ensure proper alignment of the instructional program to the Utah Core Standards for Secondary Mathematics III, Walch Education selected Amy S. Burkam, president of Lothlorien Consulting, to conduct an independent alignment study. A summary of her qualifications and experience is provided in Appendix A.

II. Methodology

The purpose of this study is to address one key question.

> To what degree does the *Common Core State Standards Integrated Pathway: Mathematics III* provide instructional materials that address the content specified by each Utah Core Standard for Secondary Mathematics III?

The criteria used in this study are adapted from the work of Dr. Norman Webb (Webb, 1997, 2005). The Webb methodology was developed to examine the alignment between assessments and standards. Webb describes four alignment criteria: Categorical Concurrence, Depth of Knowledge, Range of Knowledge, and Balance of Representation. Webb's benchmarks for meeting each criterion, and in some cases, the criterion itself, are based on the premise that assessments typically survey the content specified by the standards. For this study, Webb's criteria are adapted to serve the assertion that instruction must provide sufficient opportunities for students to master all content and skills specified by the standards.

Categorical concurrence is the degree to which standards and assessments incorporate the same content. For assessments, the categorical concurrence criterion is evaluated by determining whether the assessment includes items measuring some content from each standard. To meet the criterion for depth-of-knowledge (DoK) consistency, the cognitive processes required to answer the assessment tasks must be as demanding as the expectations defined by the standards. Typically, the DoK criterion is met for an assessment if at least 50% of the items corresponding to a standard are at or above the DoK level assigned to the performance indicator.

The range-of-knowledge criterion is used to judge whether the span of knowledge defined by a standard is comparable to the span of knowledge required to correctly answer the assessment items. Fifty percent of the objectives for a standard must have at least one related assessment item to meet this alignment criterion. Webb's range-of-knowledge criterion only considers the number of objectives within a standard assessed; it does not consider how the assessment items are distributed among the objectives. The balance-of-representation criterion is used to indicate the degree to which one objective is given more emphasis on the assessment than another.

To evaluate the alignment of the *Common Core State Standards Integrated Pathway: Mathematics III* to the Utah Core Standards for Secondary Mathematics III, this study focuses on the categorical concurrence and range-of-knowledge alignment criteria. To be considered aligned, instructional materials must provide opportunities for students to learn the material associated with every standard and every objective within the standard. The balance-of-representation criterion does not apply to the evaluation of instructional materials. If every standard and objective is thoroughly covered, any variation in emphasis is assumed to be an intentional artifact of the standards. This study does not include a formal evaluation of the cognitive demand required to complete the exercises included in the instructional materials. However, the overall rigor of exercises was noted while conducting the study and general observations are provided in the results section of this report.

The researcher compared each Utah Core Standard for Secondary Mathematics III to the content of the teacher and student resource materials. In some instances, a standard is further defined by one or more indicators (designated by a small letter after the standard number). Indicators were given the same treatment as standards. The definitions listed in Table 1 were used to assign a rating that indicates the degree to which the lessons provide opportunities for students to learn, practice, and apply the full range of knowledge and skills specified by each Utah Core Standard or indicator

Table 1
Definitions for Evaluating Strength of Alignment

Code	Description
S (Strong)	The lessons and student resources fully address the content specified by the standard (or indicator). The lessons provide sufficient opportunities for students to learn, practice, and apply the full range of knowledge and skills specified by each standard.
P (Partial)	The lessons and student resources address the content specified by the standard/indicator superficially, or cover less sophisticated skills or content than represented by the standard/indicator, or cover only a portion of the specified skills or content.
N (No Relationship)	The lessons do not address the content of the standard/indicator.

III. Findings

Strength of Alignment

Appendix B contains tables showing the alignment relationship of each Utah Core Standard or indicator to the content presented in the *Common Core State Standards Integrated Pathway: Mathematics III* instructional materials. Table 2 summarizes the findings.

Table 2

Strength of Alignment

Unit	*n*	Strength of Alignment					
		Strong		Partial		No Relationship	
		n	%	*n*	%	*n*	%
Unit 1: Inferences and Conclusions from Data							
Standards/Indicators	9	9	100%	0	0%	0	0%
Unit 2: Polynomial, Rational, and Radical Relationships							
Standards/Indicators	16	16	100%	0	0%	0	0%
Unit 3: Trigonometry of General Triangles and Trigonometric Functions							
Standards/Indicators	6	6	100%	0	0%	0	0%
Unit 4: Mathematical Modeling							
Standards/Indicators	19	19	100%	1	0%	0	0%
TOTAL	**50**	**50**	**100%**	**0**	**0%**	**0**	**0%**

Table II shows that the *Common Core State Standards Integrated Pathway: Mathematics III* instructional materials demonstrate strong alignment relationships to all standards and indicators (100%) specified by the Utah Core Standards for Secondary Mathematics III.

The researcher identified two omissions, neither of which warranted dropping the alignment relationships from “Strong” to Partial.” For standard A.APR.4, the lessons teach students to use polynomial identities, but do not explicitly prove them.

> A.APR.4 Prove polynomial identities and use them to describe numerical relationships. *For example, the polynomial identity* $(x^2 + y^2)^2 = (x^2 - y^2)^2 + (2xy)^2$ *can be used to generate Pythagorean triples.*

The instructional notes presented in Appendix A of the *Common Core State Standards for Mathematics* state that students should "use polynomial identities to solve problems.” (NGA Center and CCSSO, 2010, p. 76). The introductory paragraph to Unit 2 also omits proving polynomial identities.

> *This unit develops the structural similarities between the system of polynomials and the system of integers. Students draw on analogies between polynomial arithmetic and base-ten computation, focusing on properties of operations, particularly the distributive property. Students connect multiplication of polynomials with multiplication of multidigit integers, and division of polynomials with long division of integers. Students identify zeros of polynomials and make connections between zeros of polynomials and solutions of polynomial equations. The unit culminates with the fundamental theorem of algebra. Rational numbers extend the arithmetic of integers by allowing division by all numbers except 0. Similarly, rational expressions extend the arithmetic of polynomials by allowing division by all polynomials except the zero polynomial. A central theme of this unit is that the arithmetic of rational expressions is governed by the same rules as the arithmetic of rational numbers. (NGA Center and CCSSO, 2010, p. 74)*

In light of this paragraph, the alignment relationship of the instructional materials to standard A.APR.4 was considered "strong," and the omission of proofs was regarded as intentional and appropriate.

All aspects of standard F.IF.9 are thoroughly covered except comparing properties of functions represented graphically to functions represented in a table.

> F.IF.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.

The researcher considered this omission insufficient to justify dropping the alignment rating for standard F.IF.9 from "Strong" to "Partial."

The *Common Core State Standards Integrated Pathway: Mathematics III* instructional materials thoroughly satisfy the Common Core Standards for Mathematics III. The lessons provide ample opportunities for students to learn, practice, and apply the full range of knowledge and skills specified by each standard and indicator. The Student Resource Books and Digital Enhancements correspond directly to the lessons in the Teacher Resource Books and reflect the same level of alignment.

Depth of Knowledge

Although this study does not include a formal evaluation of the cognitive processes required of students to complete the exercises provided with the lessons, the researcher made several noteworthy observations regarding the nature and rigor of the exercises.

1. The exercises and assessment items include a variety of formats or item types.
2. Problems incorporate an assortment of real-world contexts.

3. Exercises span the range of cognitive complexity, with emphasis on application and higher order thinking skills.

The exercises include variety of formats, such as:

- multiple-choice;
- short answer (e.g., solve the problem and show your work);
- extended constructed-response, requiring the application of skills, written justification of answers, and explanation of mathematical processes; and
- performance tasks (stations), requiring hands-on engagement and exploration of mathematical concepts.

Webb defines four depth-of-knowledge levels for mathematics, with Level One being the lowest and Level Four the highest. Definitions for each level are provided in Appendix. C. In particular, the constructed response and performance activities require cognitive processes at Levels Three and Four. Students are expected to delve deeply into the content, make connections, and solve complex problems. The rigor of the exercises is consistent with the high expectations of the Utah Core Standards for Secondary Mathematics III.

Conclusion

The *Common Core State Standards Integrated Pathway: Mathematics III* instructional materials demonstrate strong alignment relationships to the content specified by the Utah Core Standards for Secondary Mathematics III. The content specified by the standards is addressed in a manner consistent with the breadth and depth indicated by the standards. The lessons provide sufficient opportunities for students to learn, practice, and apply the full range of knowledge and skills specified by each standard and indicator. The Student Resource Book and Digital Enhancements correspond directly to the lessons in the Teacher Resource and reflect the same level of alignment. The *Common Core State Standards Integrated Pathway: Mathematics III* fully meets the categorical concurrence and range-of-knowledge alignment criteria and, based on these criteria, is considered thoroughly and precisely aligned to the Utah Core Standards for Secondary Mathematics III.

References

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Appendix A

Qualifications of the Researcher

Appendix A: Qualifications of the Researcher

Assessment specialist, Amy S. Burkam, president of Lothlorien Consulting, LLC, conducted this study. A former high school science teacher, Ms. Burkam has worked in large scale assessment since 1985 when she joined the staff at Measured Progress, formerly Advanced Systems in Measurement and Evaluation, Inc. During her 22 year tenure with Measured Progress, she worked as a test developer for K-12 mathematics and science, managed and directed multiple statewide assessment programs, and directed the Curriculum and Assessment division. In her role as Curriculum and Assessment director, Ms. Burkam supervised and coordinated all aspects of item and test development. In 2007, Ms. Burkam established a private consulting firm to provide curriculum and assessment-related services to state departments of education, publishers, districts, and schools. Services include providing management and leadership for item and test development initiatives; designing and developing curricula and instructional materials; designing and conducting alignment studies; working with educators to develop content standards, item banks, assessments, and performance standards; facilitating meetings; and reviewing and editing K-12 assessment items.

Since 2007, Ms. Burkam designed and conducted the following alignment studies and predictive analyses (crosswalks).

- Alignment Analysis of Walch Education's *Common Core State Standards Integrated Pathway: Mathematics I and II* to the Common Core State Standards for Pathways II and III
- Crosswalks between WIN Learning's Career Readiness Objectives and the Common Core State Standards for Mathematics and English Language Arts
- Crosswalks between the Common Core State Standards for Mathematics and English Language Arts and the standards measured by the ERB Comprehensive Testing Program
- Alignment Analysis of the Vermont Alternate Assessment Portfolio (VTAAP) for Reading, Mathematics, and Science (March 2011)
- Alignment Analysis of Maine's 2009-2010 Personalized Alternate Assessment Portfolio (PAAP) Alternate Grade Level Expectations for Reading, Mathematics, and Science
- Alignment Analysis of the New England Common Assessment Program Expectations for Reading and Mathematics and a Form of the SAT Test of Reasoning
- Alignment Analysis of the Maine Science *Learning Results* and the Science Portion of Maine's 2009 Comprehensive Assessment System for Grades 5, 8, and 11

- Alignment Analysis of the Maine High School Mathematics *Learning Results* and a Form of the May 2009 SAT Mathematics Assessment
- Alignment Analysis of the Maine High School Reading *Learning Results* and a Form of the May 2009 SAT Reading Assessment
- Alignment Analysis of the Maine High School Mathematics *Learning Results* and a Form of the May 2008 SAT Mathematics
- Crosswalks between the Massachusetts and Tennessee reading and mathematics content standards to predict the degree of alignment between a Massachusetts item bank and the Tennessee standards
- Crosswalks between the Maine *Learning Results* for mathematics, reading, and science and the New England Common Assessment Program (NECAP) Grade Level Expectations to predict the degree of alignment between the NECAP assessment and Maine's *Learning Results*

Appendix B

Alignment Ratings

Appendix B: Alignment Ratings

***Common Core State Standards Integrated Pathway: Mathematics III Program* to the Utah Core Standards for Secondary Mathematics III**

Unit	Utah Core Standard	Lesson(s)	Teacher Resource Page #	Strength of Alignment
1	S.ID.4 Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.	1.1.1 1.1.2 1.1.3	U1-6 – 32 U1-33 – 61 U1-62 – 93	S
	S.IC.1 Understand that statistics allows inferences to be made about population parameters based on a random sample from that population.	1.2.1	U1-102 –127	S
	S.IC.2 Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. *For example, a model says a spinning coin falls heads up with probability 0. 5. Would a result of 5 tails in a row cause you to question the model?*	1.2.2 1.2.3	U1-128 –159 U1-160 – 188	S
	S.IC.3 Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.	1.3.1 1.3.2	U1-198 –215 U1-216 –234	S
	S.IC.4 Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.	1.4.1 1.4.2 1.4.3 1.4.4	U1-245 – 265 U1-266 – 292 U1-293 – 307 U1-308 – 328	S
	S.IC.5 Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant.	1.5.1 1.5.2	U1-337 – 363 U1-364 – 382	S
	S.IC.6 Evaluate reports based on data.	1.5.3	U1-383 – 399	S
	S.MD.6(+) Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator).	1.6.1	U1-408 -- 426	S
	S,MD.7 (+) Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game).	1.6.2	U1-427 – 450	S

Unit	Utah Core Standard	Lesson(s)	Teacher Resource Page #	Strength of Alignment
2	N.CN.8 (+) Extend polynomial identities to the complex numbers. For example, rewrite $x^2 + 4$ as $(x + 2i)(x – 2i)$.	2A.2.2	U2A-63 -- 75	S
	N.CN.9 (+) Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.	2A.3.3	U2A-139 -- 156	S
	A.SSE.1 Interpret expressions that represent a quantity in terms of its context.			
	A.SSE.1a Interpret parts of an expression, such as terms, factors, and coefficients.	2A.1.1 2A.2.3 2B.1.1	U2A-4 – 16 U2A-76 – 91 U2B-5 – 22	S
	A.SSE.1b Interpret complicated expressions by viewing one or more of their parts as a single entity. For example, interpret $P(1+r)^n$ as the product of P and a factor not depending on P.	2A.2.1 2A.2.2 2A.2.3 2B.1.1	U2A-46 – 62 U2A-63 – 75 U2A-76 – 91 U2B-5 – 22	S
	A.SSE.2 Interpret the structure of expressions. Use the structure of an expression to identify ways to rewrite it. For example, see $x^4 – y^4$ as $(x^2)^2 – (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 – y^2)(x^2 + y^2)$.	2A.2.1 2A.2.2 2A.2.3 2B.1.1 2B.1.2 2B.1.3 2B1.4	U2A-46 – 62 U2A-63 – 75 U2A-76 – 91 U2B-5 – 22 U2B-23 – 44 U2B-45 – 60 U2B-61 – 79	S
	A.SSE.4 Derive the formula for the sum of a geometric series (when the common ratio is not 1), and use the formula to solve problems. For example, calculate mortgage payments.	2A.5.1 2A.5.2 2A.5.3	U2A-221 – 242 U2A-243 – 270 U2A-271 -- 290	S
	A.APR.1 Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.	2A.1.2 2A.1.3	U2A-17 – 27 U2A-28 -- 39	S
	A.APR.2 Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number a, the remainder on division by $x – a$ is $p(a)$, so $p(a) = 0$ if and only if $(x – a)$ is a factor of $p(x)$.	2A.3.2	U2A-119 -- 138	S
	A.APR.3 Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.	2A.3.3 2A.3.4	U2A-139 – 156 U2A-157 -- 172	S
	A.APR.4 Prove polynomial identities and use them to describe numerical relationships. *For example, the polynomial identity* $(x^2 + y^2)^2 = (x^2 – y^2)^2 + (2xy)^2$ *can be used to generate Pythagorean triples.*	2A.2.1 2A.2.2 2A.2.3	U2A-46 – 62 U2A-63 – 75 U2A-76 -- 91	S[1]

[1] The lessons address using polynomial identities, but do not explicitly prove them.

Unit	Utah Core Standard	Lesson(s)	Teacher Resource Page #	Strength of Alignment
2	A.APR.5 (+) Know and apply the Binomial Theorem for the expansion of $(x + y)^n$ in powers of x and y for a positive integer *n*, where *x* and *y* are any numbers, with coefficients determined for example by Pascal's Triangle.	2A.2.3	U2A-76 -- 91	S
	A.APR.6 Rewrite simple rational expressions in different forms; write *a(x)/b(x)* in the form *q(x)* + *r(x)/b(x)*, where *a(x), b(x), q(x),* and *r(x)* are polynomials with the degree of *r(x)* less than the degree of *b(x)*, using inspection, long division, or, for the more complicated examples, a computer algebra system.	2B.1.4	U2B-61 – 79	S
	APR.7 (+) Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.	2B.1.2 2B.1.3 2B.1.4	U2B-23 – 44 U2B-45 – 60 U2B-61 – 79	S
	A.REI.2 Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.	2B.2.1 2B.2.2	U2B-87 – 106 U2B-107 – 126	S
	A.REI.11 Explain why the *x*-coordinates of the points where the graphs of the equations *y* = *f(x)* and *y* = *g(x)* intersect are the solutions of the equation f(x) = g(x); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where *f(x)* and/or *g(x)* are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.	2A.4.1 2B.2.3	U2A-182 – 209 U2B-127 – 155	S
	F.IF.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.			
	F.IF.7c Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.	2A.3.1 2A.3.3	U2A-100 – 118 U2A-139 -- 156	S

Unit	Utah Core Standard	Lesson(s)	Teacher Resource Page #	Strength of Alignment
3	G.SRT.9 (+) Derive the formula $A = (1/2)ab \sin(C)$ for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.	3.2.1	U3-101 – 124	S
	G.SRT.10 (+) Prove the Laws of Sines and Cosines and use them to solve problems.	3.2.1 3.2.2	U3-101 – 124 U3-125 – 143	S
	G.SRT.11 (+) Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).	3.2.3	U3-144 – 161	S
	F.TF.1 Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.	3.1.1	U3-5 – 21	S
	F.TF.2 Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.	3.1.1 3.1.2 3.1.3 3.1.4	U3-5 – 21 U3-22 – 45 U3-46 – 69 U3-70 – 92	S
	F.TF.5 Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.	3.3.1 3.3.2	U3-169 – 186 U3-187 – 205	S

Unit	Utah Core Standard	Lesson(s)	Teacher Resource Page #	Strength of Alignment
4	A.CED.1 Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.	4B.1.1	U4B-7 – 35	S
	A.CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.	4B.4.1	U4B-248 – 272	S
	A.CED.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.	4B.1.2	U4B-36 – 53	S
	A.CED.4 Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. For example, rearrange Ohm's law V = IR to highlight resistance R.	4B.1.3	U4B-54 – 81	S
	F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.	4A.2.5 4B.3.1 4B.4.1 4B.4.2 4B.4.3	U4A-164 – 182 U4B-155 – 189 U4B-248 – 272 U4B-273 – 296 U4B-297 – 319	S
	F.IF.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.	4A.2.5 4B.3.1 4B.4.1 4B.4.2 4B.4.3	U4A-164 – 182 U4B-155 – 189 U4B-248 – 272 U4B-273 – 296 U4B-297 – 319	S
	F.IF.6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.	4A.2.5 4B.3.1 4B.3.2 4B.3.3	U4A-164 – 182 U4B-155 – 189 U4B-190 – 211 U4B-212 – 234	S
	F.IF.7 Graph functions expressed symbolically and show key Features of the graph, by hand in simple cases and using technology for more complicated cases.			
	F.IF.7b Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.	4B.4.2 4B.4.3	U4B-273 – 296 U4B-297 – 319	S

Unit	Utah Core Standard	Lesson(s)	Teacher Resource Page #	Strength of Alignment
4	F.IF.7e Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.	4A.2.4 4A.3.1 4A.3.2	U4A-138 – 163 U4A-191 – 223 U4A-234 – 272	S
	F.IF.8 Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.	4A.2.2 4A.2.3	U4A-98 –15 U4A-116 – 137	S
	F.IF.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.	4B.3.3	U4B-212 – 234	S[2]
	F.BF.1 Write a function that describes a relationship between two quantities.			
	F.BF.1b Combine standard function types using arithmetic operations. For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.	4B.2.3	U4B-131 – 144	S
	F.BF.3 Build new functions from existing functions. Identify the effect on the graph of replacing *f(x)* by *f(x)* + *k*, *k f(x)*, *f(kx)*, and *f*(*x* + *k*) for specific values of *k* (both positive and negative); find the value of *k* given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.	4B.2.1 4B.2.2 4B.4.1 4B.4.2 4B.4.3	U4B-90 – 115 U4B-116 – 130 U4B-248 – 272 U4B-273 – 296 U4B-297 – 319	S
	F.BF.4 Build new functions from existing functions. Find inverse functions.			
	F.BF.4a Solve an equation of the form *f(x)* = *c* for a simple function *f* that has an inverse and write an expression for the inverse. For example, $f(x) = 2(x^3)$ or *f(x)* = (*x*+1)/(*x*-1) for *x* ≠ 1.	4A.1.1 4A.1.2 4A.2.1	U4A-5 – 34 U4A-35 – 63 U4A-73 – 97	S
	F.LE.4 For exponential models, express as a logarithm the solution to $ab^{(ct)} = d$ where *a, c*, and *d* are numbers and the base *b* is 2, 10, or e; evaluate the logarithm using technology.	4A.2.1 4A.2.2 4A.2.3	U4A-73 – 97 U4A-98 –15 U4A-116 – 137	S

[2] All aspects of standard F.IF.9 are thoroughly covered except comparing properties of functions represented graphically to functions represented in a table.

Unit	Utah Core Standard	Lesson(s)	Teacher Resource Page #	Strength of Alignment
4	G.GMD.4 Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.	4B.5.1	U4B-273 – 296	S
	G.MG.1 Apply geometric concepts in modeling situations. Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).	4B.5.1	U4B-273 – 296	S
	G.MG.2 Apply geometric concepts in modeling situations. Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).	4B.5.2	U4B-350 – 369	S
	G.MG.3 Apply geometric concepts in modeling situations. Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).	4B.5.3	U4B-370 – 392	S

Appendix C

Depth of Knowledge Definitions for Mathematics

Appendix C: Mathematics Depth of Knowledge Definitions

(Dr. Norman Webb)

Level 1 (Recall) includes the recall of information such as a fact, definition, term, or a simple procedure, as well as performing a simple algorithm or applying a formula. A one-step, well-defined, algorithmic procedure is considered a Level 1 activity. Keywords that signify Level 1 exercises include "identify," "recall," "recognize," "use," and "measure." Verbs such as "describe" and "explain" could be classified at different levels, depending on what the student must describe and explain.

Level 2 (Skill/Concept) involves the engagement of some mental processing beyond a habitual response. A Level 2 assessment item requires students to decide how to approach the problem or activity, whereas a Level 1 item requires students to provide a rote response, perform a memorized algorithm, follow a set procedure (like a recipe), or perform a clearly defined series of steps. Keywords or phrases that generally distinguish a Level 2 item include "classify," "organize," "estimate," "make observations," "collect and display data," and "compare data." These actions imply more than one step. For example, to compare data a student must identify characteristics of the objects or phenomenon and then group or order the objects. Verbs such as "explain," "describe," or "interpret," could be classified at different levels depending on the object of the action. Interpreting information from a simple graph is a Level 2 task. Interpreting information from a complex graph that requires determining which features of the graph need to be considered or how information from the graph can be aggregated is at Level 3. Level 2 activities are not limited to number skills, but can involve visualization skills and probability skills. Other Level 2 activities include noticing and describing non-trivial patterns, explaining the purpose and use of experimental procedures; carrying out experimental procedures; making observations and collecting data; classifying, organizing, and comparing data; and organizing and displaying data in tables, graphs, and charts.

Level 3 (Strategic Thinking) requires reasoning, planning, using evidence, and a higher level of thinking than the previous two levels. In most instances, exercises that require students to explain their thinking are at Level 3. Activities that require students to make conjectures are also at this level. The cognitive demands at Level 3 are complex and abstract. The complexity does not result from the fact that there are multiple answers, a possibility for both Levels 1 and 2, but because the task requires more demanding reasoning. An activity that has more than one possible answer *and* requires students to justify their responses would most likely be at Level 3. Other Level 3 activities include drawing conclusions from observations; citing evidence and developing a logical argument for concepts; explaining phenomena in terms of concepts; and using concepts to solve problems.

Level 4 (Extended Thinking) requires complex reasoning, planning, developing, and thinking, most likely over an extended period of time. The extended time period is not a distinguishing factor if the work is repetitive and does not require significant conceptual understanding and higher-order thinking. For example, an activity that involves measuring the water temperature of a river each day for a month and then constructing a graph would be classified at Level 2. However, if the student must conduct a river study that requires consideration of several variables, the task is likely at Level 4. At Level 4, the cognitive demands of the task should be high and the work should be complex. Students should be required to make several connections—relate ideas *within* the content area or *among* content areas—and select one approach among many alternatives. Level 4 activities include developing and proving conjectures; designing and conducting experiments; making connections between a finding and related concepts and phenomena; combining and synthesizing ideas into new concepts; and critiquing experimental designs.