Alignment Analysis of the
Walch Education Coordinate Algebra Program to the
Common Core Georgia Performance Standards
for Coordinate Algebra

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Amy S. Burkam, President
Lothlorien Consulting, LLC
369 Crew Road
Wakefield, New Hampshire 03872
Phone: 603-522-3123 e-mail: aburkam@roadrunner.com

This study was conducted for Walch Education, Portland, ME.

Table of Contents

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I. Introduction

In a standards based education system, alignment between expectations for student learning, instruction, and assessment is critical. Alignment expert, Dr. Norman Webb, defines alignment as is the degree to which the various components of an educational system—expectations, curricula, instruction, and assessments—are in agreement and work together to achieve desired goals for student achievement. Close alignment helps educators focus on the desired content and ensures that students have a fair opportunity to learn and to demonstrate their knowledge and understanding. (Webb, 1997, 2005)

The Coordinate Algebra Program is a comprehensive set of instructional materials developed by Walch Education specifically to address the *Common Core Georgia Performance Standards* (CCGPS) *for Coordinate Algebra*. To verify the alignment of the instructional program to the CCGPS for Coordinate Algebra, Walch Education selected Amy S. Burkam, president of Lothlorien Consulting, to conduct an independent alignment study. A summary of her qualifications and experience is provided in Appendix A.

II. Methodology

The purpose of this study is to address one key question.

> To what degree does the Walch Education Coordinate Algebra Program provide instructional materials that address the content specified by each CCGPS for Coordinate Algebra?

The criteria used in this study are adapted from the work of Dr. Norman Webb (Webb, 1997, 2005). The Webb methodology was developed to examine the alignment between assessments and standards. Webb describes four alignment criteria: Categorical Concurrence, Depth of Knowledge, Range of Knowledge, and Balance of Representation. Webb's benchmarks for meeting each criterion, and in some cases, the criterion itself, are based on the premise that assessments typically survey the content specified by the standards. For this study, Webb's criteria are adapted to serve the assertion that instruction must provide sufficient opportunities for students to master all content and skills specified by the standards.

Categorical concurrence is the degree to which standards and assessments incorporate the same content. For assessments, the categorical concurrence criterion is evaluated by determining whether the assessment includes items measuring some content from each standard. To meet the criterion for depth-of-knowledge (DoK) consistency, the cognitive processes required to answer the assessment tasks must be as demanding as the expectations defined by the standards. Typically, the DoK criterion is met for an assessment if at least 50% of the items corresponding to a standard are at or above the DoK level assigned to the performance indicator. The range-of-knowledge criterion is used to judge whether the span of knowledge defined by a standard is comparable to the span of knowledge required to correctly answer the assessment items. Fifty percent of the objectives for a standard must have at least one related assessment item to meet this alignment criterion. Webb's range-of-knowledge criterion only considers the number of objectives within a standard assessed; it does not consider how the assessment items are distributed among the objectives. The balance-of-representation criterion is used to indicate the degree to which one objective is given more emphasis on the assessment than another.

To evaluate the alignment of the Walch Education Coordinate Algebra program to the CCGPS for Coordinate Algebra, this study focuses on the categorical concurrence and range-of-knowledge alignment criteria. To be considered aligned, instructional materials must provide opportunities for students to learn the material associated with every standard and every objective within the standard. The balance-of-representation criterion does not apply to the evaluation of instructional materials. If every standard and objective is thoroughly covered, any variation in emphasis is assumed to be an intentional artifact of the standards. This study does not include a formal evaluation of the cognitive demand required to complete the exercises included in the instructional materials. However, the overall rigor of exercises was noted while conducting the study and general observations are provided in the results section of this report.

To conduct the study, the researcher compared each CCGPS for Coordinate Algebra to the content of the teacher and student resource materials to determine the degree to which the lessons provide opportunities for students to learn, practice, and apply the full range of knowledge and skills specified by each standard. In instances where the CCGPS are further elaborated by indicators, the content of the resource materials was also evaluated against each indicator. The definitions listed in Table 1 were applied to each CCGPS.

Table 1

Definitions for Evaluating Strength of Alignment

Code	Description
S (Strong)	The lesson(s) and student materials fully address the content specified by the standard (or indicators below the standard, when present). The lessons provide sufficient opportunities for students to learn, practice, and apply the full range of knowledge and skills specified by each standard.
P (Partial)	The lesson(s) and student materials address the content specified by the standard/indicator superficially, or cover less sophisticated skills or content than represented by the standard/indicator, or cover only a portion of the specified skills or content.
N (No Relationship)	The lesson(s) and student materials do not address the content of the standard/indicator.

III. Findings

Strength of Alignment

Appendix B contains tables showing the alignment relationship of each CCGPS to the Walch Education Coordinate Algebra Program. Table 2 summarizes the findings.

Table 2

Strength of Alignment (CCGPS to the Walsh Education Coordinate Algebra Program)

Unit	*n*	Strength of Alignment					
		Strong		Partial		No Relationship	
		N	%	*N*	%	*n*	%
1. Relationship between Quantities							
Standards	8	8	100%	0	0%	0	0%
Indicators	2	2	100%	0	0%	0	0%
2. Reasoning with Equations and Inequalities							
Standards	5	5	100%	0	0%	0	0%
Indicators	0	0		0	0%	0	0%
3. Linear & Exponential Functions							
Standards	17	17	100%	0	0%	0	0%
Indicators	7	7	100%	0	0%	0	0%
4. Describing Data							
Standards	8	8	100%	0	0%	0	0%
Indicators	3	3	100%	0	0%	0	0%
Transition Standards	1	1	100%	0	0%	0	0%
5. Transformations in the Coordinate Plane							
Standards	5	5	100%	0	0%	0	0%
Indicators	0	0		0	0%	0	0%
6. Connecting Algebra & Geometry through Coordinates							
Standards	4	4	100%	0	0%	0	0%
Indicators	0	0		0	0%	0	0%
Transition Standards	1	1	100%	0	0%	0	0%

The Walch Education Coordinate Algebra Program has a strong alignment relationship to all content and skills (100%) specified by the CCGPS. The lessons provide ample opportunities for students to learn, practice, and apply the full range of knowledge and skills specified by each standard and indicator. The Student Resource Book and Digital Enhancements correspond directly to the lessons in the Teacher Resource books and reflect the same level of alignment.

Depth of Knowledge

Although this study does not include a formal evaluation of the cognitive processes required of students to complete the exercises provided with the lessons, the researcher made several noteworthy observations regarding the nature and rigor of the exercises.

1. The exercises and assessment items include a variety of formats or item types.
2. Problems incorporate a variety of real-world contexts.
3. Exercises span the range of cognitive complexity, with emphasis on application and higher order thinking skills.

The exercises include variety of formats, such as:

- multiple-choice;
- short answer (e.g., graph the equation or solve the problem and show your work);
- extended constructed-response , requiring the application of skills and written explanations of the mathematical processes; and
- performance tasks, requiring hands-on engagement and exploration of mathematical concepts.

Webb defines four depth-of-knowledge levels for mathematics, with Level One being the lowest and Level Four the highest. Definitions for each level are provided in Appendix. C. In particular, the constructed response and performance activities require cognitive processes at Levels Three and Four. Students are expected to delve deeply into the content, make connections, and solve complex problems.

Conclusion

The Walch Education Coordinate Algebra Program demonstrates a strong alignment relationship to the content specified by the CCGPS for Coordinate Algebra. The lessons provide opportunities for students to learn, practice, and apply the full range of knowledge and skills specified by each standard and indicator. The Student Resource Book and Digital Enhancements correspond directly to the lessons in the Teacher Resource books and reflect the same level of alignment. The Walch Education Coordinate Algebra Program meets the categorical concurrence and range-of-knowledge alignment criteria and, based on these criteria, is considered accurately and thoroughly aligned to the CCGPS for Coordinate Algebra.

References

Webb, N.L. (1997). Criteria for alignment of expectations and assessments in mathematics and science education. (NISE Research and Monograph No. 8). Madison: University of Wisconsin—Madison, National Institute for Science Education. Washington DC: Council of Chief State School Officers.

Webb, N.L. (2005). *Web Alignment Tool (WAT) Training Manual Draft Version 1.1.* Madison: University of Wisconsin, Wisconsin Center for Education Research.

Appendix A

Qualifications of the Researcher

Appendix A: Qualifications of the Researcher

Assessment specialist, Amy S. Burkam, president of Lothlorien Consulting, LLC, conducted this study. A former high school science teacher, Ms. Burkam has worked in large scale assessment since 1985 when she joined the staff at Measured Progress, formerly Advanced Systems in Measurement and Evaluation, Inc. During her 22 year tenure with Measured Progress, she worked as a test developer for K-12 mathematics and science, managed and directed multiple statewide assessment programs, and directed the Curriculum and Assessment division. In her role as Curriculum and Assessment director, Ms. Burkam supervised and coordinated all aspects of item and test development. In 2007, Ms. Burkam established a private consulting firm to provide assessment-related services to state departments of education, assessment companies, districts, and schools. Services include providing management and leadership for item and test development initiatives; designing and conducting alignment studies; working with educators to develop content standards, item banks, assessments, and performance standards; facilitating meetings; and reviewing and editing K-12 assessment items.

Since 2007, Ms. Burkam designed and conducted the following alignment studies and predictive analyses (crosswalks).

- Crosswalks between WIN Learning's Career Readiness Objectives and the Common Core State Standards for Mathematics and English Language Arts
- Crosswalks between the Common Core State Standards for Mathematics and English Language Arts and the standards measured by the ERB Comprehensive Testing Program
- Alignment Analysis of the Vermont Alternate Assessment Portfolio (VTAAP) for Reading, Mathematics, and Science (March 2011)
- Alignment Analysis of Maine's 2009-2010 Personalized Alternate Assessment Portfolio (PAAP) Alternate Grade Level Expectations for Reading, Mathematics, and Science
- Alignment Analysis of the New England Common Assessment Program Expectations for Reading and Mathematics and a Form of the SAT Test of Reasoning
- Alignment Analysis of the Maine Science *Learning Results* and the Science Portion of Maine's 2009 Comprehensive Assessment System for Grades 5, 8, and 11
- Alignment Analysis of the Maine High School Mathematics *Learning Results* and a Form of the May 2009 SAT Mathematics Assessment
- Alignment Analysis of the Maine High School Reading *Learning Results* and a Form of the May 2009 SAT Reading Assessment

- Alignment Analysis of the Maine High School Mathematics *Learning Results* and a Form of the May 2008 SAT Mathematics
- Crosswalks between the Massachusetts and Tennessee reading and mathematics content standards to predict the degree of alignment between a Massachusetts item bank and the Tennessee standards
- Crosswalks between the Maine *Learning Results* for mathematics, reading, and science and the New England Common Assessment Program (NECAP) Grade Level Expectations to predict the degree of alignment between the NECAP assessment and Maine's *Learning Results*

Appendix B

Strength of Alignment Ratings

Appendix B: Strength of Alignment Ratings

Unit	CCGPS	Lesson(s)	Teacher Resource Page #	Strength of Alignment
1	**MCC9-12.N.Q.1** Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.	1.3.1 1.3.2	U1-95-150	S
	MCC9-12.N.Q.2 Define appropriate quantities for the purpose of descriptive modeling.	1.2.1	U1-35-55	S
	MCC9-12.N.Q.3 Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.	1.2.1	U1-35-55	S
	MCC9-12.A.SSE.1 Interpret expressions that represent a quantity in terms of its context. *(Emphasis on linear expressions and exponential expressions with integer exponents.)*	1.1.1 1.2.1	U1-5-20	S
	a. Interpret parts of an expression, such as terms, factors, and coefficients. *(Emphasis on linear expressions and exponential expressions with integer exponents.)*	1.1.1	U1-5-16	S
	b. Interpret complicated expressions by viewing one or more of their parts as a single entity. *(Emphasis on linear expressions and exponential expressions with integer exponents.)*	1.1.2	U1-17-22	S
	MCC9-12.A.CED.1 Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear ~~and quadratic functions, and simple rational~~ and exponential functions.	1.2.1 1.2.2 1.2.3	U1-35-86	S
	MCC9-12.A.CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. *(Limit to linear and exponential equations, and, in the case of exponential equations, limit to situations requiring evaluation of exponential functions at integer inputs.)*	1.3.1 1.3.2	U1-95-150	S
	MCC9-12.A.CED.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. *(Limit to linear equations and inequalities.)*	1.3.3	U1-168-182	S
	MCC9-12.A.CED.4 Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. (*Limit to formulas with a linear focus.)*	1.5.1	U1-188-199	S

Unit	CCGPS	Lesson(s)	Teacher Resource Page #	Strength of Alignment
2	**MCC9-12.A.REI.1** Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method. *(Students should focus on and master linear equations and be able to extend and apply their reasoning to other types of equations in future courses.)*	2.1.1	U2-3-18	S
	MCC9-12.A.REI.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters. *(Extend earlier work with solving linear equations to solving linear inequalities in one variable and to solving literal equations that are linear in the variable being solved for. Include simple exponential equations that rely only on application of the laws of exponents, such as 5x = 125 or 2x = 1/16.)*	2.1.2 2.1.3 2.1.4	U2-19-60	S
	MCC9-12.A.REI.5 Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions. *(Limit to linear equations.)*	2.2.1	U2-70-87	S
	MCC9-12.A.REI.6 Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.	2.2.2	U2-89-112	S
	MCC9-12.A.REI.12 Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.	2.3.1 2.3.2	U2-120-164	S

Unit	CCGPS	Lesson(s)	Teacher Resource Page #	Strength of Alignment
3	**MCC9-12.A.REI.10** Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line). *(Focus on linear and exponential equations and be able to adapt and apply that learning to other types of equations in future courses.)*	3.1.1	U3-4-21	S
	MCC9-12.A.REI.11 Explain why the *x*-coordinates of the points where the graphs of the equations *y = f(x)* and *y = g(x)* intersect are the solutions of the equation *f(x) = g(x)*; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where *f(x)* and/or *g(x)* are linear, ~~polynomial, rational, absolute value,~~ exponential, ~~and logarithmic~~ functions.	3.1.2	U3-22-36	S
	MCC9-12.F.IF.1 Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If *f* is a function and x is an element of its domain, then *f(x)* denotes the output of *f* corresponding to the input *x*. The graph of f is the graph of the equation *y = f(x)*. *(Draw examples from linear and exponential functions.)*	3.1.3	U3-37-56	S
	MCC9-12.F.IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. *(Draw examples from linear and exponential functions.)*	3.1.4	U3-57-70	S
	MCC9-12.F.IF.3 Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. *(Draw connection to F.BF.2, which requires students to write arithmetic and geometric sequences.)*	3.2.1	U3-79-98	S
	MCC9-12.F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and ~~periodicity~~. *(Focus on linear and exponential functions.)*	3.3.1	U3-108-133	S

Unit	CCGPS	Lesson(s)	Teacher Resource Page #	Strength of Alignment
3	**MCC9-12.F.IF.5** Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *(Focus on linear and exponential functions.)*	3.3.1	U3-108-133	S
	MCC9-12.F.IF.6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. *(Focus on linear functions and intervals for exponential functions whose domain is a subset of the integers.)*	3.3.2 3.3.3	U3-134-169	S
	MCC9-12.F.IF.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. *(Focus on linear and exponential functions. Include comparisons of two functions presented algebraically.)*			S
	a. Graph linear ~~and quadratic~~ functions and show intercepts, maxima, and minima.	3.4.1	U3-184-202	S
	e. Graph exponential ~~and logarithmic~~ functions, showing intercepts and end behavior~~, and trigonometric functions, showing period, midline, and amplitude.~~	3.4.2	U3-203-222	S
	MCC9-12.F.IF.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *(Focus on linear and exponential functions. Include comparisons of two functions presented algebraically.)*	3.5.1 3.5.2	U3-235-280	S
	MCC9-12.F.BF.1 Write a function that describes a relationship between two quantities. *(Limit to linear and exponential functions.)*			
	a. Determine an explicit expression, a recursive process, or steps for calculation from a context. *(Limit to linear and exponential functions.)*	3.6.1	U3-310-329	S
	b. Combine standard function types using arithmetic operations. *(Limit to linear and exponential functions.)*	3.7.1	U3-360-370	S
	MCC9-12.F.BF.2 Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.	3.8.1 3.8.2	U3-395-416	S

Unit	CCGPS	Lesson(s)	Teacher Resource Page #	Strength of Alignment
	MCC9-12.F.BF.3 Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. *(Focus on vertical translations of graphs of linear and exponential functions. Relate the vertical translation of a linear function to its y-intercept.)*	3.7.2	U3-371-386	S
	MCC9-12.F.LE.1 Distinguish between situations that can be modeled with linear functions and with exponential functions.			
	a. Prove that linear functions grow by equal differences over equal intervals; exponential functions grow by equal factors over equal intervals.	3.3.2	U3-134-150	S
3	b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.	3.3.3	U3-151-169	S
	c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.	3.3.3	U3-151-169	S
	MCC9-12.F.LE.2 Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).	3.6.2	U3-330-350	S
	MCC9-12.F.LE.3 Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, ~~quadratically, or (more generally) as a polynomial function~~.	3.5.3	U3-281-298	S
	MCC9-12.F.LE.5 Interpret the parameters in a linear or exponential function in terms of a context. *(Limit exponential functions to those of the form f(x) = bx + k.)*	3.9.1	U3-422-437	S

Unit	CCGPS	Lesson(s)	Teacher Resource Page #	Strength of Alignment
4	**MCC9-12.S.ID.1** Represent data with plots on the real number line (dot plots, histograms, and box plots).	4.1.2	U4-25-46	S
	MCC9-12.S.ID.2 Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, ~~standard deviation~~) of two or more different data sets. *(Standard deviation is left for Advanced Algebra, use MAD as a measure of spread.)*	4.1.3	U4-47-67	S
	MCC9-12.S.ID.3 Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).	4.1.4	U4-68-86	S
	MCC9-12.S.ID.5 Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.	4.2.1	U4-96-113	S
	MCC9-12.S.ID.6 Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.			S
	a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear, ~~quadratic,~~ and exponential models.	4.2.2	U4-114-136	S
	b. Informally assess the fit of a function by plotting and analyzing residuals.	4.2.3	U4-137-160	S
	c. Fit a linear function for scatter plots that suggest a linear association.	4.2.4	U4-161-183	S
	MCC9-12.S.ID.7 Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.	4.3.1	U4-195-215	S
	MCC9-12.S.ID.8 Compute (using technology) and interpret the correlation coefficient of a linear fit.	4.3.2	U-216-235	S
	MCC9-12.S.ID.9 Distinguish between correlation and causation.	4.3.3	U4-236-258	S
	Transition Standard: **MCC6.SP.5** Summarize numerical data sets in relation to their context, such as by: c. Giving quantitative measures of ~~center (median and/or mean) and~~ variability (~~interquartile range and/or~~ mean absolute deviation), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data was gathered.	4.1.1	U4-7-24	S

Unit	CCGPS	Lesson(s)	Teacher Resource Page #	Strength of Alignment
5	**MCC9-12.G.CO.1** Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.	5.1.1	U5-5-17	S
	MCC9-12.G.CO.2 Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).	5.1.2	U5-18-37	S
	MCC9-12.G.CO.3 Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.	5.1.3	U5-38-54	S
	MCC9-12.G.CO.4 Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.	5.2.1	U5-60-71	S
	MCC9-12.G.CO.5 Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.	5.2.2	U5-72-85	S
6	**MCC9-12.G.GPE.4** Use coordinates to prove simple geometric theorems algebraically. *(Restrict contexts that use distance and slope.)*	6.1.2	U6-26-53	S
	MCC9-12.G.GPE.5 Prove the slope criteria for parallel and perpendicular lines; use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).	6.1.2 6.1.3	U6-26-74	S
	MCC9-12.MCC9-12.G.GPE.6 Find the point on a directed line segment between two given points that partitions the segment in a given ratio.	6.2.1	U6-82-104	S
	MCC9-12.G.GPE.7 Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.«	6.2.2	U6-105-129	S
	Transition Standard: **MCC8.G.8** Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.	6.1.1	U6-4-25	S

Appendix C

Depth of Knowledge Definitions for Mathematics

Appendix C: Mathematics Depth of Knowledge Definitions

(Dr. Norman Webb)

Level 1 (Recall) includes the recall of information such as a fact, definition, term, or a simple procedure, as well as performing a simple algorithm or applying a formula. A one-step, well-defined, algorithmic procedure is considered a Level 1 activity. Keywords that signify Level 1 exercises include “identify,” “recall,” “recognize,” “use,” and “measure.” Verbs such as “describe” and “explain” could be classified at different levels, depending on what the student must describe and explain.

Level 2 (Skill/Concept) involves the engagement of some mental processing beyond a habitual response. A Level 2 assessment item requires students to decide how to approach the problem or activity, whereas a Level 1 item requires students to provide a rote response, perform a memorized algorithm, follow a set procedure (like a recipe), or perform a clearly defined series of steps. Keywords or phrases that generally distinguish a Level 2 item include “classify,” “organize,” ”estimate,” “make observations,” “collect and display data,” and “compare data.” These actions imply more than one step. For example, to compare data a student must identify characteristics of the objects or phenomenon and then group or order the objects. Verbs such as “explain,” “describe,” or “interpret,” could be classified at different levels depending on the object of the action. Interpreting information from a simple graph is a Level 2 task. Interpreting information from a complex graph that requires determining which features of the graph need to be considered or how information from the graph can be aggregated is at Level 3. Level 2 activities are not limited to number skills, but can involve visualization skills and probability skills. Other Level 2 activities include noticing and describing non-trivial patterns, explaining the purpose and use of experimental procedures; carrying out experimental procedures; making observations and collecting data; classifying, organizing, and comparing data; and organizing and displaying data in tables, graphs, and charts.

Level 3 (Strategic Thinking) requires reasoning, planning, using evidence, and a higher level of thinking than the previous two levels. In most instances, exercises that require students to explain their thinking are at Level 3. Activities that require students to make conjectures are also at this level. The cognitive demands at Level 3 are complex and abstract. The complexity does not result from the fact that there are multiple answers, a possibility for both Levels 1 and 2, but because the task requires more demanding reasoning. An activity that has more than one possible answer *and* requires students to justify their responses would most likely be at Level 3. Other Level 3 activities include drawing conclusions from observations; citing evidence and developing a logical argument for concepts; explaining phenomena in terms of concepts; and using concepts to solve problems.

Level 4 (Extended Thinking) requires complex reasoning, planning, developing, and thinking, most likely over an extended period of time. The extended time period is not a distinguishing factor if the work is repetitive and does not require significant conceptual understanding and higher-order thinking. For example, an activity that involves measuring the water temperature of a river each day for a month and then constructing a graph would be classified at Level 2. However, if the student must conduct a river study that requires consideration of several variables, the task is likely at Level 4. At Level 4, the cognitive demands of the task should be high and the work should be complex. Students should be required to make several connections—relate ideas *within* the content area or *among* content areas—and select one approach among many alternatives. Level 4 activities include developing and proving conjectures; designing and conducting experiments; making connections between a finding and related concepts and phenomena; combining and synthesizing ideas into new concepts; and critiquing experimental designs.